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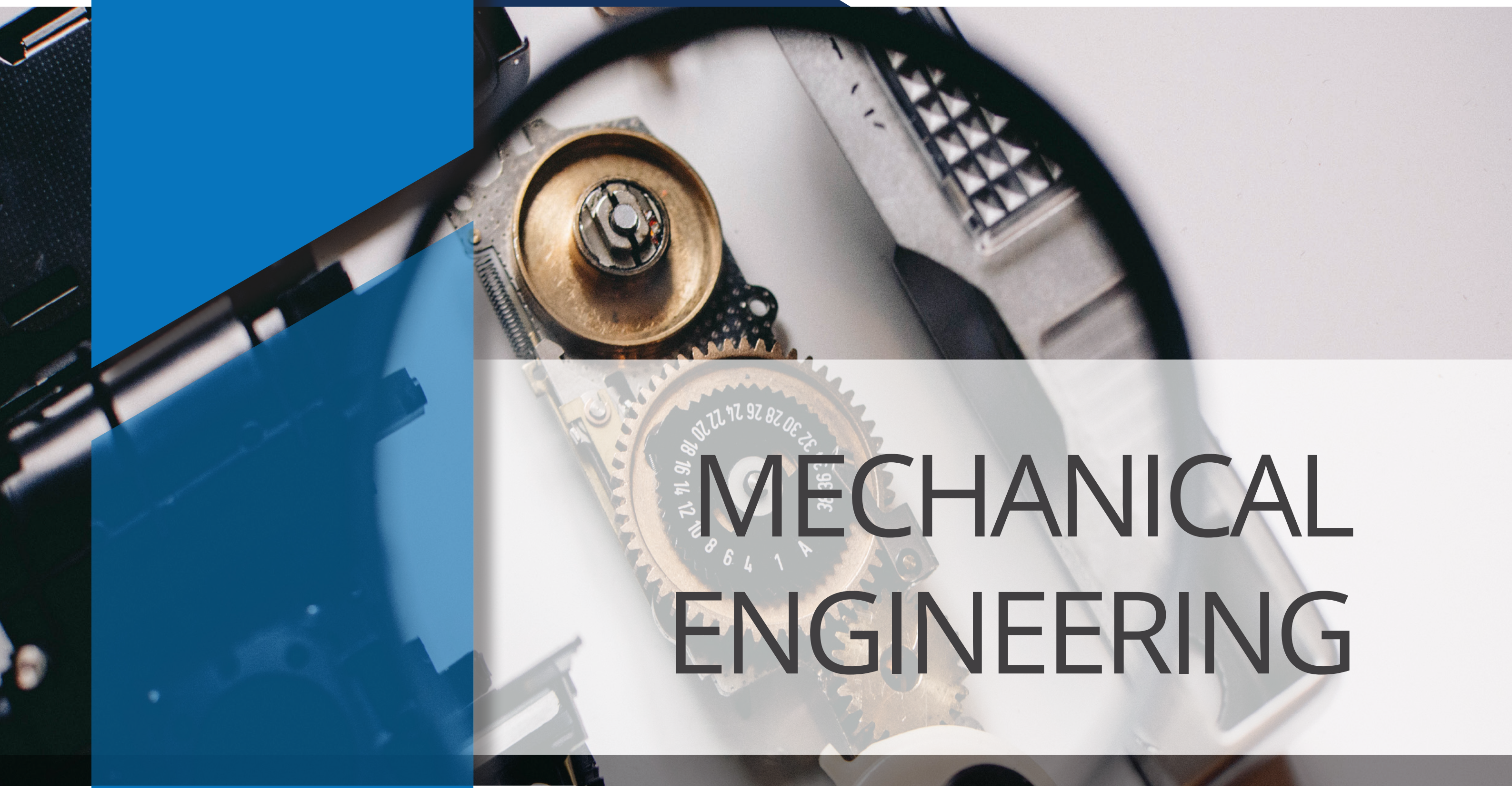


MECHANICAL ENGINEERING

Statics

UNIVERSITY OF APPLIED SCIENCES UPPER AUSTRIA

VŠTE

EUROPEAN UNION

Contents

1. BASIC STATICS TERMS, PRINCIPLES AND AXIOMS

1.1. Basic terms

Rigid body

It refers to a perfectly rigid body. It is a figure, body where the distance between any two randomly selected points does not change due to the movement of the body.

Particle

Physical object whose dimensions are negligible. In statics, point it refers to the point of a rigid in which all the weight of the entire rigid is concentrated.

Force and moment

Force is a basic measure of two objects mutual effect. Force is a vector quantity. Force is a vector linked to a line.

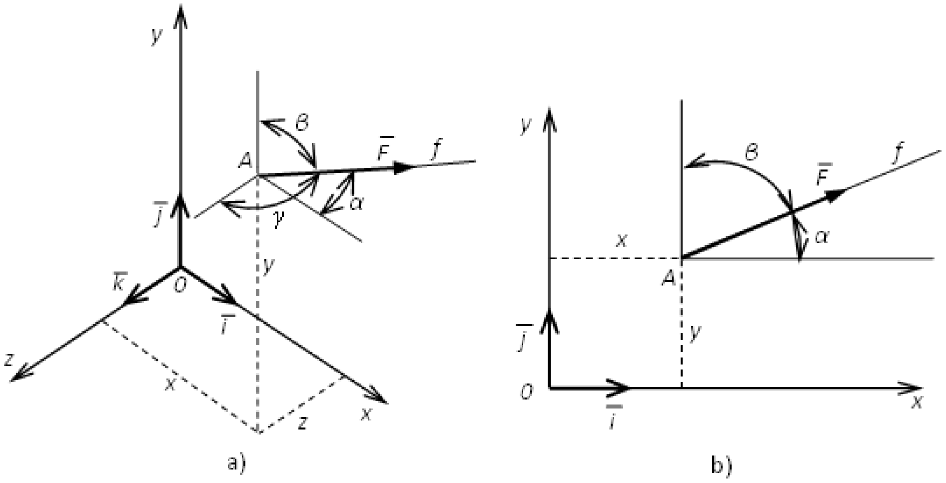


Figure 1.1

- Force in space (*Figure 1.1a*) is determined by 6 parameters: point of application A(x, y, z) – 3 parameters, size F – 1 parameter, position f and direction – angles α, β, (γ) – 2 independent parameters, since the angles are mutually linkeded as follows:

$cos^2\alpha + cos^2\beta + cos^2\gamma = 1.$

- Force in plane (*Figure 1.1b*) is determined by 4 parameters: A(x, y), F, α (β), since

$$\alpha + \beta = \frac{\pi}{2}$$
$$a\cos^2\alpha + \cos^2\beta = 1.$$

Force effect

- Sliding – identical for all points of the object on which the force is acting. It is equal to the force,

- Rotational – different for different points of the object. The size of the rotational effect depends on the perpendicular distance between the point and the force bearer and is determined by the moment to the relevant point.

Moment – is a vector defined as a vector product $\underline{M} = \underline{r} \times \underline{F}$**.** Absolute size of moment to any point, e.g. A equals to the product of force and the moment arm – its perpendicular distance from this point (*Figure 1.2*).

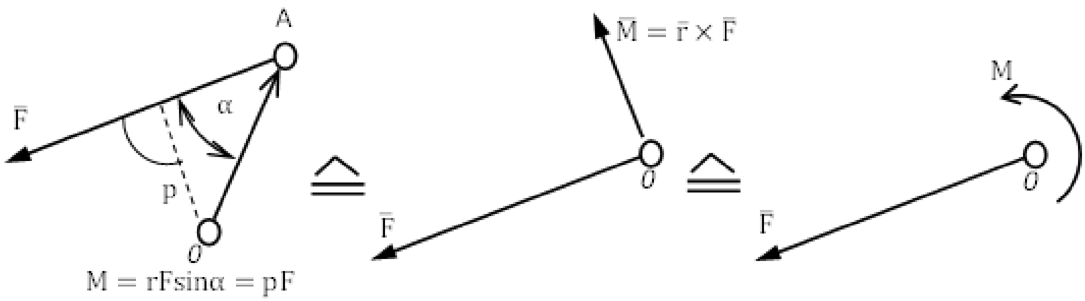


Figure 1.2

The direction of the moment vector is given by the direction of the rotation of the force $\underline{F}$ to the point A. Moment is positive if the rotation is anticlockwise.

Moment vector is represented as a vector perpendicular to the plane of rotation. The direction of the vector is determined using the right hand rule (fingers of right hand show the direction of the rotation, thumb showing the direction of the vector $\underline{M}$ (*Figure 1.3*).

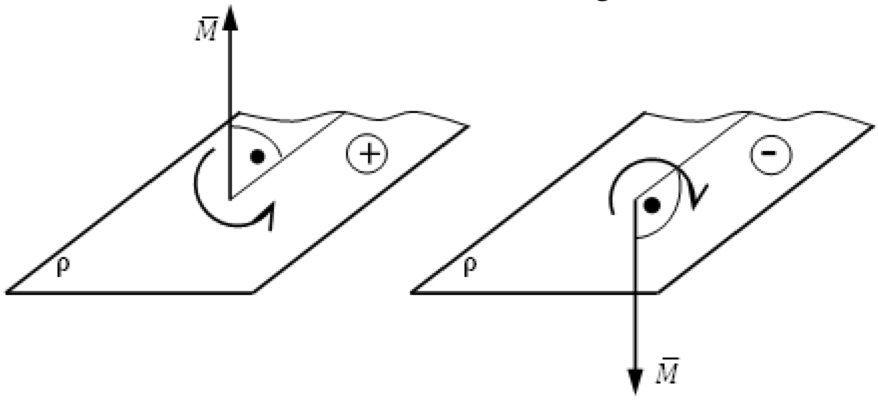


Figure 1.3

Force couple (pure moment) – two parallel forces, equal in magnitude x with different line of action. Force couple does not have sliding effect, only a rotational effect equal to the product of one force and the perpendicular distance between the forces. The effect of the force couple is given by its moment (Figure 1.4)

$$M = p\,F.$$

Vector of force couple momentum $\underline{M}$ is a free vector, which means it could be freely moved in the space and is perpendicular to the plane of force couple action.

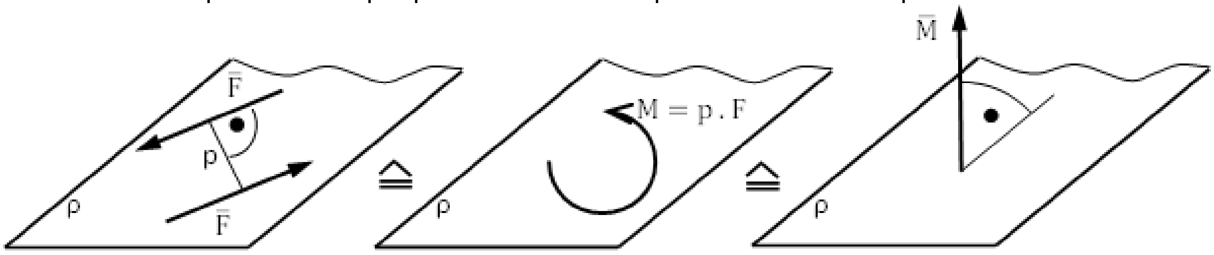


Figure 1.4

1.2. Force and moment units

The force unit is 1 Newton [N]. It is a force producing acceleration of 1ms-2 to 1 kg of weight, that is

$$1N = 1kg.m.s^{-2}.$$

The moment unit is 1N.m [Nm]

1.3. Force systems

Two and more forces acting on one object create a force system.

- If a force system can be replaced by one force $\underline{R}$, this force is called the force system resultant. A force system has a sliding effect in the direction of the resultant $\underline{R}$ bearer (Figure 1.5).

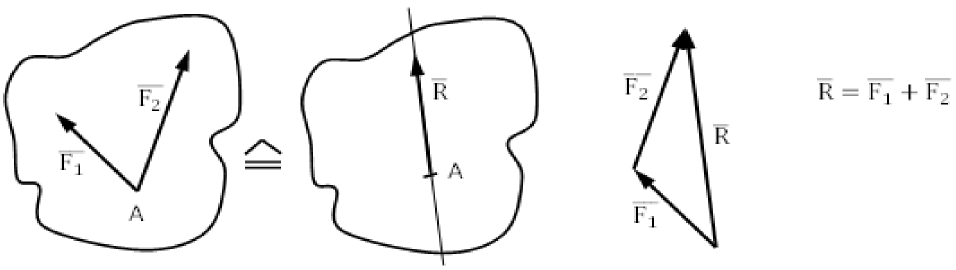


Figure 1.5

- If a force system can be replaced by one moment $\underline{M_v}$, this moment is called resulting moment of the given force system. The force system thus has a rotational effect in the plane perpendicular to the moment $\underline{M_v}$ (Figure 1.6).

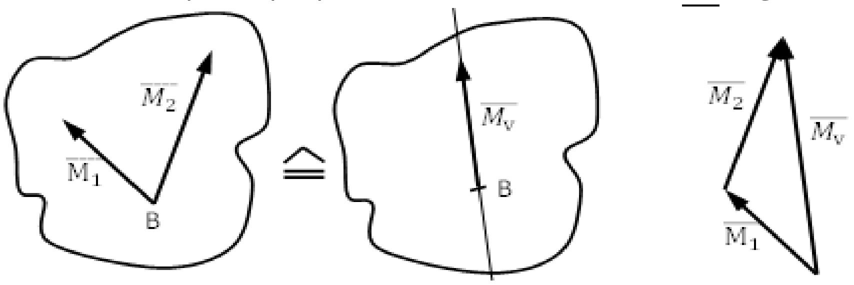
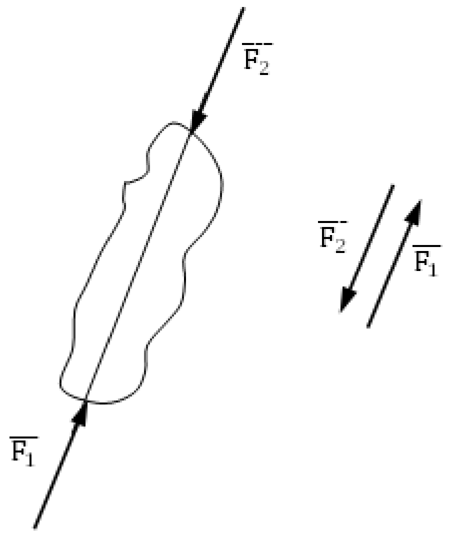


Figure 1.6

- In general, force system has both sliding and rotational effect.
- Force system is balanced if the resulting sliding and rotational effect is zero. The simplest balanced force system consists of two forces on one bearer, which are of equal magnitude and different direction (Figure 1.7).



$$\overline{F_1} = -\overline{F_2}$$

$$\overline{F_1} + \overline{F_2} = \bar{0}$$

Figure 1.7

1.4. Classification of forces by acting

- External forces (Figure 1.8a), representing the effect of the surrounding objects on the examined object. These include the loading forces - primary ($\underline{F}$) and binding reactions – secondary ($\underline{A}$), depending on the loading forces,

- internal (Figure 1.8b), representing the effect of one part of a rigid (system) on another one ($\underline{N_1}$, $\underline{N_1}'$). Internal forces are caused by external forces.

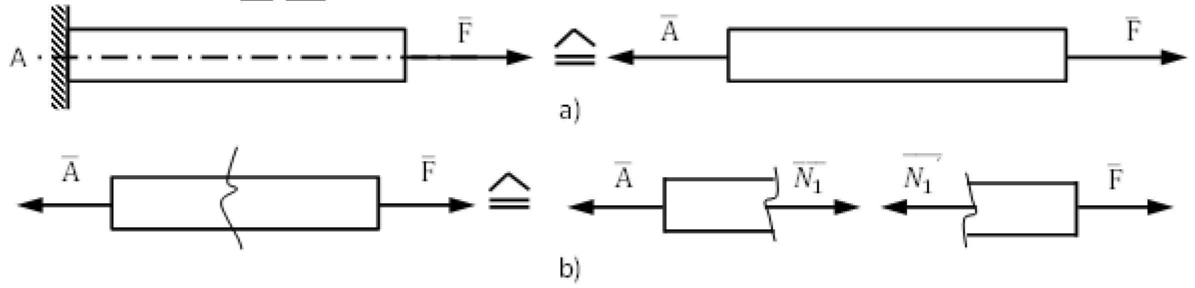


Figure 1.8

1.5. Decomposition of force, force components

Force $\underline{F}$ in at a given point in a plane can be divided into two components $\underline{F_1}$, $\underline{F_2}$. If a force $\underline{F}$ and directions of its components bearer are given (determined by angles α, β), we can determine the magnitude of the components $\underline{F_1}$, $\underline{F_2}$ (Figure 1.9). If a force $\underline{F}$ is a resultant of the forces $\underline{F_1}$ and $\underline{F_2}$, then $\underline{F} = \underline{F_1} + \underline{F_2}$.

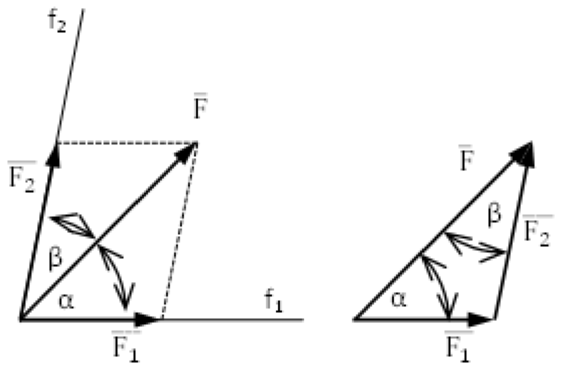


Figure 1.9

In the space, the forces can be divided into three components (Figure 1.10a).

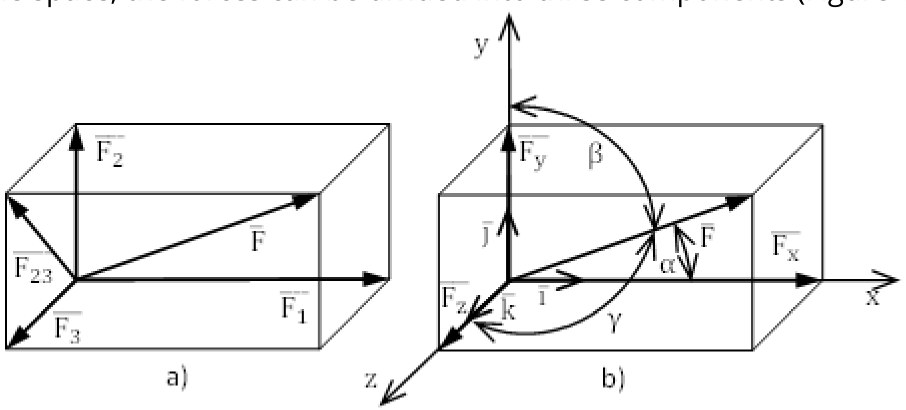


Figure 1.10

$\underline{F} = \underline{F_1} + \underline{F_{23}}$
$\underline{F_{23}} = \underline{F_2} + \underline{F_3}$
$\underline{F} = \underline{F_1} + \underline{F_2} + \underline{F_3}$

If the force components are mutually perpendicular, coordinate axes can be placed in their directions. They will be marked $\underline{F_x}$, $\underline{F_y}$, $\underline{F_z}$ and called right-angled components of the given force (Figure 1.10b).

$\underline{F} = \underline{F_x} + \underline{F_y} + \underline{F_z}$

In a rectangular coordinate system, the magnitudes of the components are as follows:

$F_x = \underline{F}.\underline{i} = F \cos \cos \alpha$
$F_y = \underline{F}.\underline{j} = F \cos \cos \beta$ $\underline{i}, \underline{j}, \underline{k}$ – unit vectors
$F_z = \underline{F}.\underline{k} = F \cos \cos \gamma$

Components can be expressed as follows:
$\underline{F_x} = F_x \underline{i}$
$\underline{F_y} = F_y \underline{j}$
$\underline{F_z} = F_z \underline{k}$

Resulting force is described as follows:
$\underline{F} = F_x \underline{i} + F_y \underline{j} + F_z \underline{k}$
If we know the magnitude of the force components, its magnitude can be calculated as follows:
$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

And its direction can be determined by means of angles α, β, (γ):
$\cos \cos \alpha = \frac{F_x}{F}$, $= \frac{F_y}{F}$, $\cos \cos \gamma = \frac{F_z}{F}$.

1.5.1. Varignon´s theorem

Force moment to a given point equals to the sum of components moments to the same point. According to Figure 1.11 the moment of force $\underline{F}$ to point 0 is $\underline{M_0} = \underline{r} \times \underline{F}$.

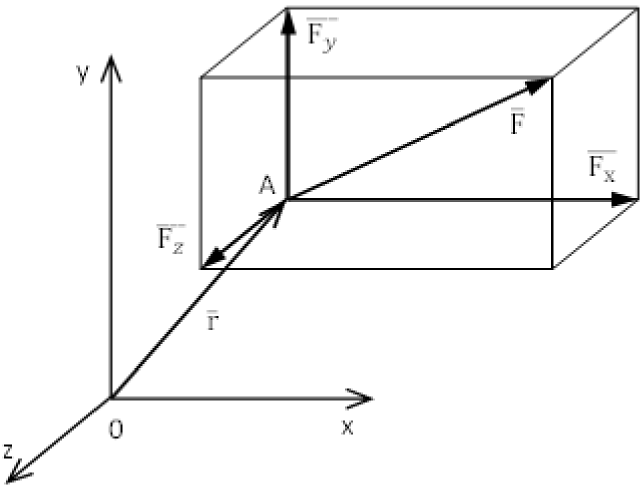


Figure 1.11

Moment of force $\underline{F}$ to point 0 is $\underline{M_0} = \underline{r} \times \underline{F}$, because

$\underline{F} = \underline{F_x} + \underline{F_y} + \underline{F_z}$,

$\underline{M_0} = \underline{r} \times \left(\underline{F_x} + \underline{F_y} + \underline{F_z}\right)$,

$\underline{M_0} = \underline{r} \times \underline{F_x} + \underline{r} \times \underline{F_y} + \underline{r} \times \underline{F_z}$, a teda

$\underline{M_0} = \underline{M_{0Fx}} + \underline{M_{0Fy}} + \underline{M_{0Fz}}$.

1.6. Basic static principles and axioms

Axiom is a basic proposition that is accepted without evidence. Usually it is based on experimental experience.

Classical mechanics is based on three basic Newton laws:

- Law of inertia (1st Newton law)
- Law of force (2nd Newton law)
- Law of action-reaction (3rd Newton law)

Statics is based on the following axioms:

1.6.1. Axiom of inertia (1st Newton law)

An object which is at rest or uniform direct motion remains in this state if no external force acts on it or if a balanced force system acts on it.

1.6.2. Axiom of action-reaction (3rd Newton law)

For every action, there is a reaction of the same magnitude and different direction. This means that the effect of one object to another is the same as the effect of the second object to the first, but with different direction (Figure 1.12).

$$\underline{F_{12}} + \underline{F_{21}} = 0$$
$$\underline{F_{12}} = -\underline{F_{21}}$$

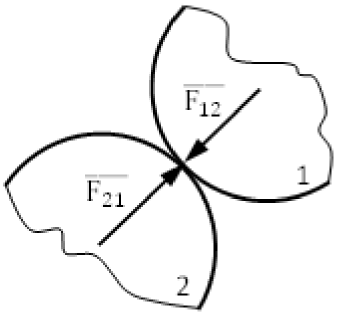


Figure 1.12

1.6.3. Axiom of preserving effect

The effect of a given force system does not change if a balanced force system is added or removed.

1.6.4. Axiom of forces composing

The resultant $\underline{R}$ of two concurrent forces $\underline{F_1}$ a $\underline{F_2}$ equals to their vectors sum $\underline{R} = \underline{F_1} + \underline{F_2}$ and passes through the point of intersection of their lines of action (Figure 1.13).

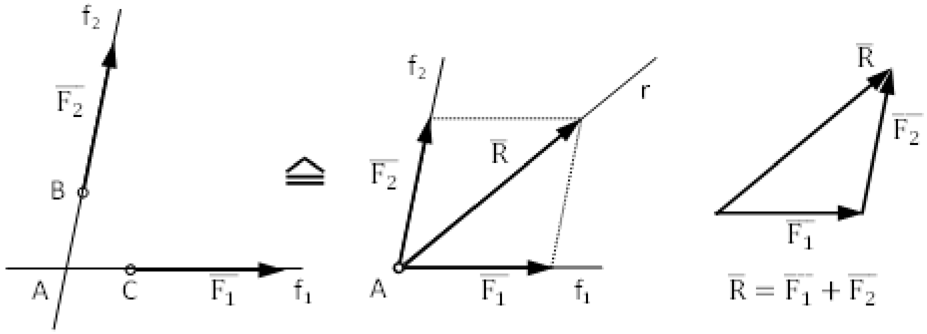


Figure 1.13

Example 1

At the beginning of the selected coordinate system 0, x, y, force $\underline{F}$ of magnitude F = 6kN (Figure 1.1.1) is acting. The direction of the line of action of the force $\underline{F}$ is given by the angle α = 30°. Divide the force $\underline{F}$ into its components $\underline{F_x}$, $\underline{F_y}$, whose lines of action f_x, f_y are identical with the coordinate system axes x, y.

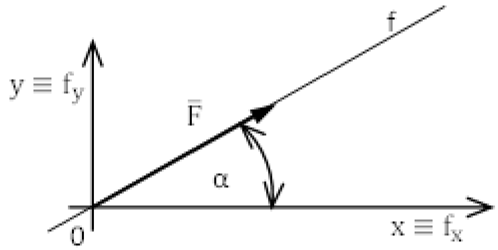


Figure 1.1.1

Solution: Dividing the force $\underline{F}$, which lies on the line of action f, whose compositions are replacements by the equivalent force system $\underline{F_x}$, $\underline{F_y}$, lying on the lines of action f_x, f_y. If the lines of action f_x, f_y are identical with the coordinate system axes x, y, then forces $\underline{F_x}$, $\underline{F_y}$ are coordinate components of force $\underline{F}$ in a given coordinate system. The solution is based on the vector substitution condition (a), where the division of force $\underline{F}$ can be done in several ways.

$$\underline{F} = \underline{F_x} + \underline{F_y} \qquad \text{(a)}$$

Analytical solution:

- Solution by means of directional angles cosines α, β of the $\underline{F}$ line of action of the force with the line of action of the force fx, fy (Figure 1.1.2)

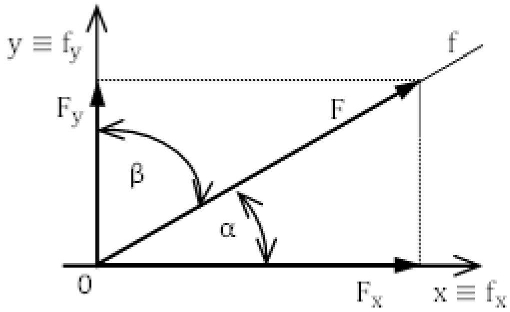


Figure 1.1.2

$$\beta = 90° - \alpha = 90° - 30° = 60°$$
$$F_x = F \ \cos\cos\alpha \ = 6 \cos\cos 30° \ = 5{,}196kN$$
$$F_y = F \ \cos\cos\beta \ = 6 \cos\cos 60° \ = 3kN$$

- Another solution is using trigonometric relations of a right triangle. According to Figure 1.1.3 for magnitudes $\underline{F}$, $\underline{F_x}$, $\underline{F_y}$ it holds true that:

$$cos\, cos\, \alpha \ = \frac{F_x}{F} \Rightarrow \ F_x = F\, cos\, cos\, \alpha \ = 6\, cos\, cos\, 30° \ = 5{,}196kN$$
$$cos\, cos\, \alpha \ = \frac{F}{F_y} \Rightarrow \ F_y = F\, sin\, sin\, \alpha \ = 6\, sin\, sin\, 30° \ = 3kN$$

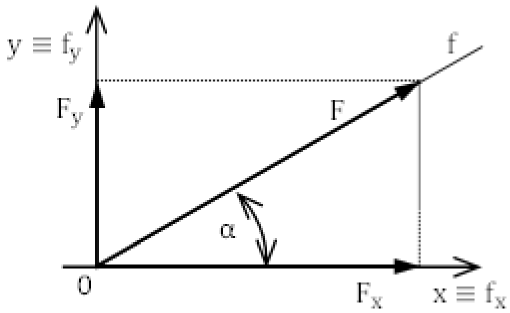


Figure 1.1.3

Graphic solution: Force $\underline{F}$ can be replaced by the equivalent force system $\underline{F_x}$, $\underline{F_y}$. It is a graphical sum of force vectors in so-called free-body diagram, where force $\underline{F}$ as a resultant of two concurrent forces with a common point of application, applied to a free-body diagram in any order, is an oriented line starting from the starting point of the first force and entering the end point of the second force.

The solution is based on Figure 1.1.4a, where we draw the know force $\underline{F}$ and the parameters of the sought forces. In the case of forces $\underline{F_x}$, $\underline{F_y}$ their lines of action of the force are known. In this figure, it is important to draw correctly the vectors of the individual forces (their directions). Free-body diagram (Figure 1.1.4b) will be created as follows:

We apply force $\underline{F}$ on the parallel with the line of action of the force f in appropriately chosen force magnitudes m_F. The starting and end point of the force $\underline{F}$ vector are parallel lines with lines of action of the force f_x, f_y in any order. The result is a closed triangle, whose legs represent the magnitudes of the forces sought. The direction of these forces in the free-body diagram is opposite to the direction of their resultant $\underline{F}$. By measuring the length of the graphical representations of the forces sought and their comparing with the force measure generates the actual magnitude of these forces.

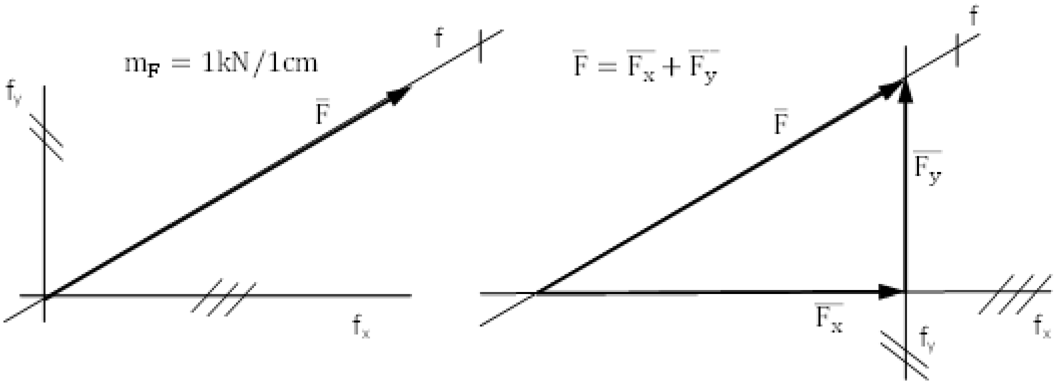


Figure 1.1.4

$F_{xg} = 5{,}2cm \Rightarrow F_x = F_{xg}m_F = 5{,}2kN$
$F_{yg} = 3cm \Rightarrow F_y = F_{yg}m_F = 3kN$

2. MOBILITY AND LINKS OF MATERIAL OBJECTS

2.1. Links and links dependency

Force systems act on specific material object (particle, rigid body, system of rigid bodies, system of particles). Material objects can be placed freely in plane or in space, that is, with an unlimited possibility to move, or are linked by links that reduce their ability to move. In these links forces are generated – so called links reactions.

The links by which the system is attached to a stationary rigid body – so-called frame, are external links and reactions arising in them are external reactions (Figure 2.1).

If a mechanical system (system of rigid bodies, particles) consists of several objects, the links between them are inner links and reactions in them are internal reactions.

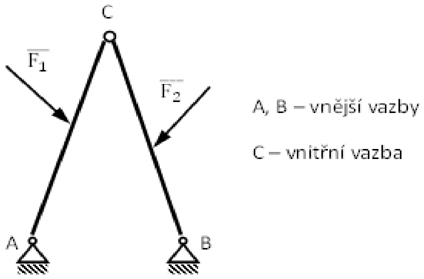


Figure 2.1

The links reduce the mobility of material objects, while the individual types of links can prevent only certain object movements. The binding reactions (secondary forces) induced by object loading by external, loading (primary) forces, they can act only in the direction in which the links are able to prevent the movement.

If the link prevents the movement only on one side, it is referred to as a unilateral binding (or force). Figure 2.2a shows an example of link by one-sided bracing, Figure 2.2b shows an example of a rope link.

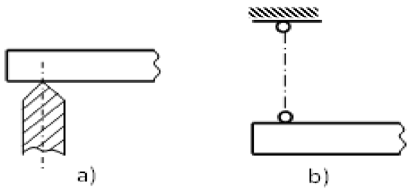


Figure 2.2

If the link prevents the movement on both sides, it is referred to as a bilateral link. Figure 2.3a shows an example of a double-sided binding, Figure 2.3b shows an example of vazby prutem.

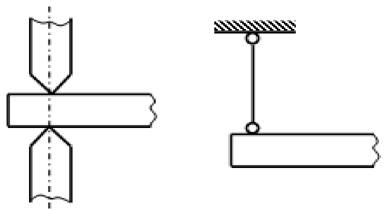


Figure 2.3

2.2. Degrees of freedom of movement and link dependency of rigid bodies

The number of degrees of degrees of freedom is a number of all independent parameters, which determine the object position in a plane or space. It also expresses the number of possible movements the given object can execute in a plane or space.

Mobility or immobility (kinematic determinacy) of a material object is assessed by its link dependency:

$i = v - u$

where:i – is a number of degrees of freedom of a material object
v – is a number of degrees of freedom of a free, not linked object
u – is a number of degrees of freedom removed by links

2.3. Kinematic and static determinacy

When assessing kinematic determinacy, i.e. mobility or immobility of a material object, the link dependency is expressed as follows:

- $i = v - u = 0$, the task is kinematically determinate. Links reduce all possibilities of movement, the position of the object is predetermined.
- $i = v - u > 0$, the task is kinematically indeterminate. Links reduce less degrees of freedom of a free object. The position of an object can change.
- $i = v - u < 0$, the task is kinematically overdetermined. Links reduce more degrees of freedom, its position is overdetermined.

By analysing static determinacy it is possible to assess whether there is a sufficient number of conditions, i.e. conditions for equilibrium to determine the unknown parameters of link reactions. Since $v = r$; $u = n_p$, it holds true that aj $i = i_S$, we can simultaneously assess both kinematic and static determinacy of tasks:

<table>
<tr><td></td><td>></td><td>the task is kinematically indeterminate, statically over-determined</td></tr>
<tr><td>$i = i_S = v - u = r - n_p$</td><td>= 0</td><td>the task is kinematically and statically determined</td></tr>
<tr><td></td><td><</td><td>the task is kinematically overdetermined, statically in-determinate</td></tr>
</table>

where: i_S – degree of static determinacy

r – number of independent conditions of equilibrium

n_p – number of unknown parameters of link reactions

Statically indeterminate ÚLOHA cannot be solved only by using static methods. Such tasks are solved in terms of flexibility and strength, which determine other, so-called deformation conditions.

3. PARTICLE IN PLANE

3.1. Degree of freedom and link dependence of particle in plane

The position of free particle M in the plane determined by the coordinate system 0, x, y (Figure 2.4) is determined by two independent parameters x_M, y_M. Particle thus has two degrees of freedom of movement $v = 2$, that is, is able to perform two independent movements (placement of axes x and y) and link dependence is:

$$i = v - u = 2 - u \quad \begin{matrix} > \\ = \\ < \end{matrix} \; 0$$

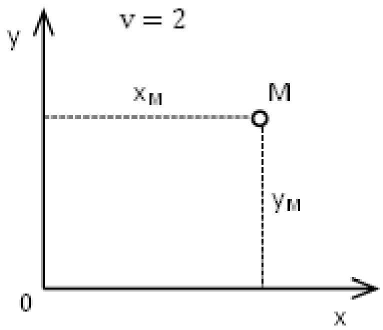


Figure 2.5

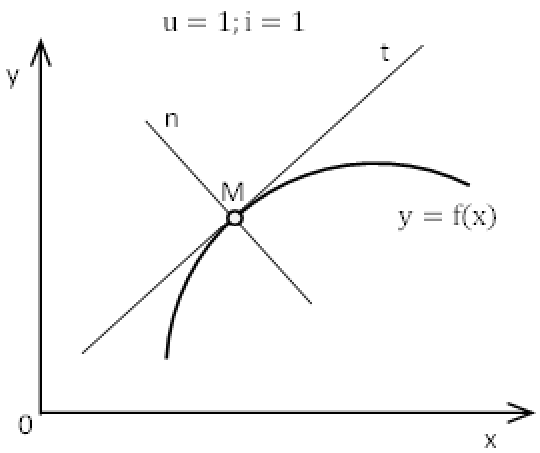


Figure 2.4

3.2. Links of particle in plane

Particle in the fix plane to the curve $y = f(x)$ can move only tangentially (see Figure 2.5). The movement in the direction of the normal is reduced, therefore the possible link reactions in particle loading is a normal reaction. The position of the particle is determined by one data, e.g. by coordinatex_M; $[y_M = f(x_M)]$; therefore the particle linked to planar curve has at least one degree of freedom of movement.

$$u = 1; i = 2 - u = 2 - 1 = 1$$

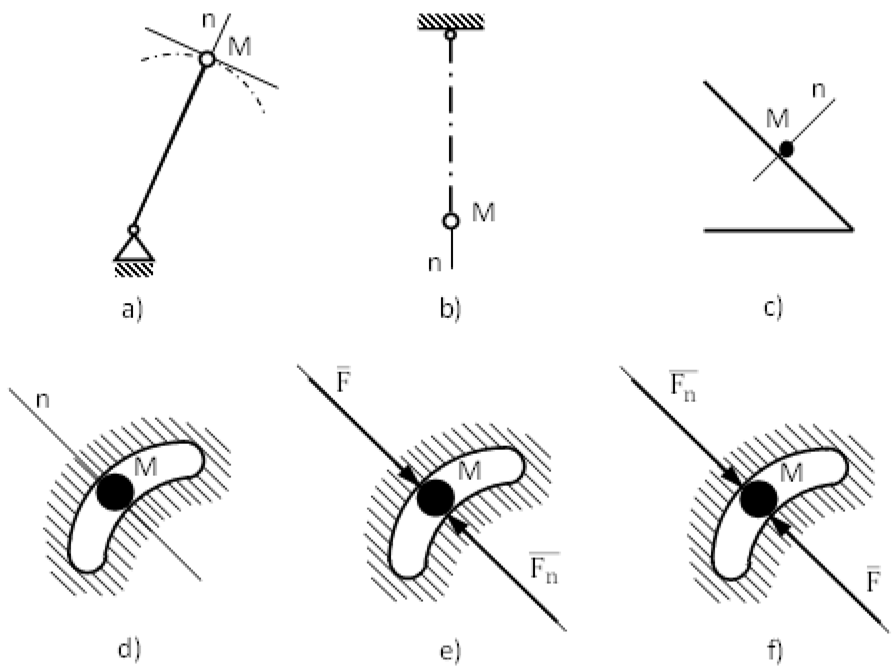


Figure 2.6

A particle can be linked to one planar curve by a bar (Figure 2.6a), or groove, e.g. sliding block (kámen v kulise) (Figure 2.6d). Such VAZBY are bilateral (forced), and can produce a reaction to both directions (Figure 2.6e,f). Rope links (Figure 2.6b) and linkage by leaning (Figure 2.6c) take away one degree of freedom of movement from the particle in the plane, but only on one side. These are referred to as unilateral (force) links.

Particle linked to two curves at the same time (Figure 2.7a) has been taken 2 degrees of freedom of movement and has no possibility to move. Such a link can be realized e.g. by two bars (Figure 2.7b).

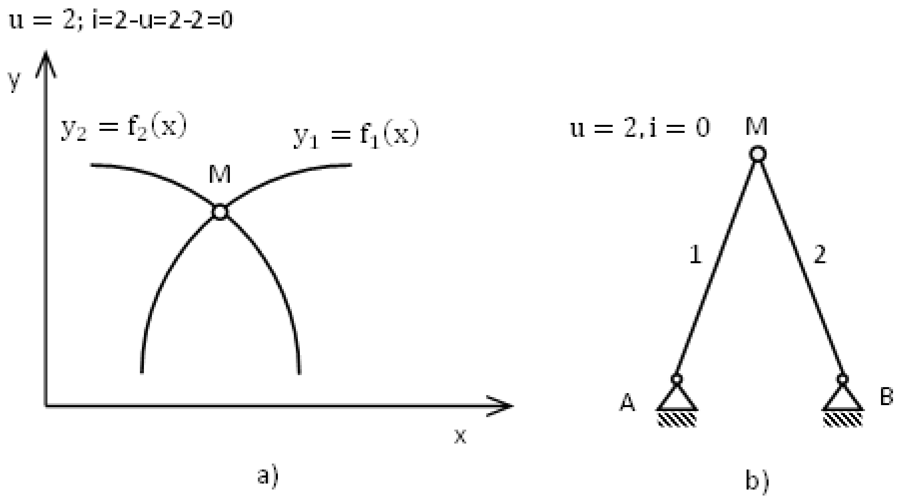


Figure 2.7

3.3. Particle in space

3.3.1. Degrees of freedom and link dependence of particle in space

The position of a free particle M in the space is determined by 3 parameters. In orthogonal coordinate system 0, x, y, z these are the following three coordinates: x_M, y_M, z_M (Figure 2.8). A free particle has three degrees of freedom of movement in the space ($v = 3$), which means it can perform three independent movements (moving in the direction of the axis x, y, z) and its link dependence is as follows:

$$i = v - u = 3 - u \quad \begin{matrix} > \\ = \\ < \end{matrix} \; 0$$

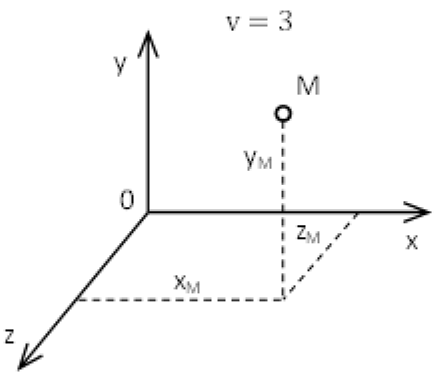


Figure 2.8

3.3.2. Links of particle in space

A rigid body in space can be linked to surface (Figure 2.9a), e.g. by a bar or a rope to a round surface (Figure2.9b), placing on a plane (Figure 2.9c) etc. This way we take away one degree of freedom of movement in the direction of the normal to the surface, or in the direction of a rod.

$u = 1, i = 3 - u = 3 - 1 = 2$

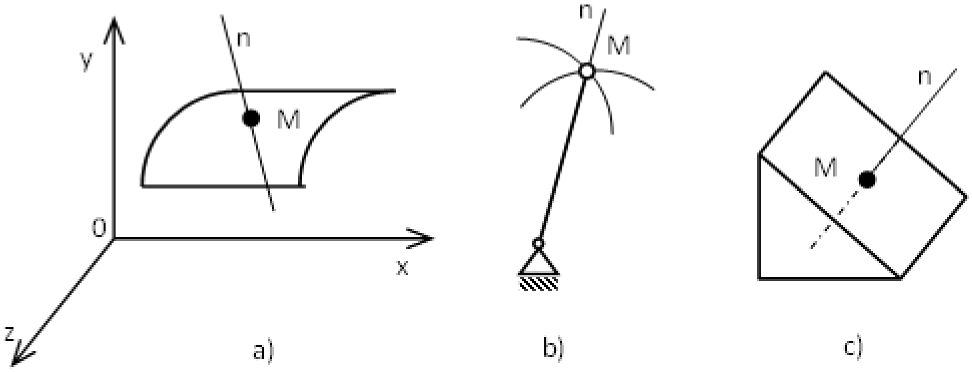


Figure 2.9

If we connect a particle in the space to two surfaces (Figure 2.10a), we will take away two degrees of freedom of movement. It is actually connected in the intersection, i.e. to the spatial curve. A realization of this link is e.g. by two bars (Figure 2.10b).

$u = 2, i = 3 - u = 3 - 2 = 1$

By connecting a particle to three surfaces that intersect in a given point, all three degrees of freedom are removed. This is e.g. link by three bars that must not lie in the same plane. They musts create so-called bearing block (Figure 2.11).

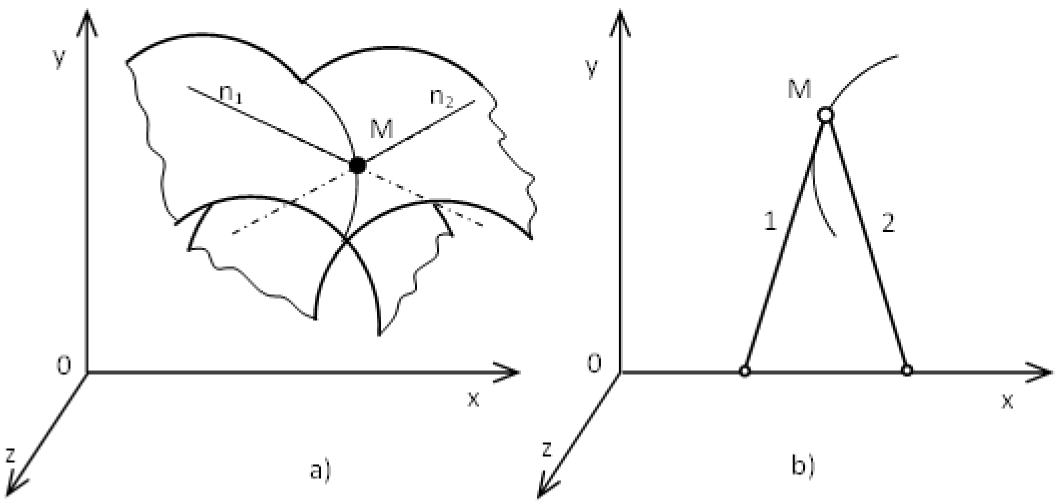


Figure 2.10

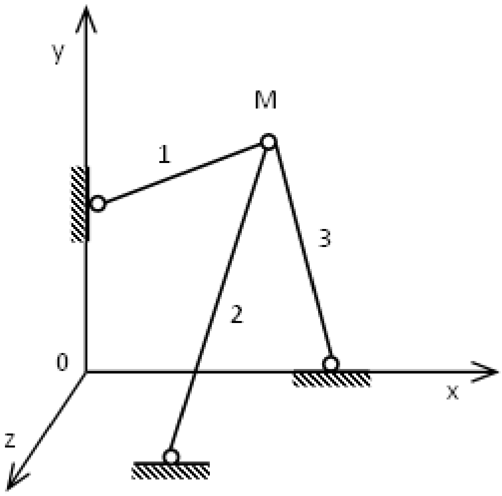


Figure 2.11

3.4. Rigid body in plane

3.4.1. Degrees of freedom of rigid body in plane

The position of a rigid in the plane (0, x, y) is determined by three parameters. These can be coordinates of point A (x_A, y_A) and angle φ of the line joining A and B (Figure 2.12), or two coordinates of point A and one coordinate of point B. The second coordinate of point B is given by the fact, that the distance $\underline{AB}$ is constant.

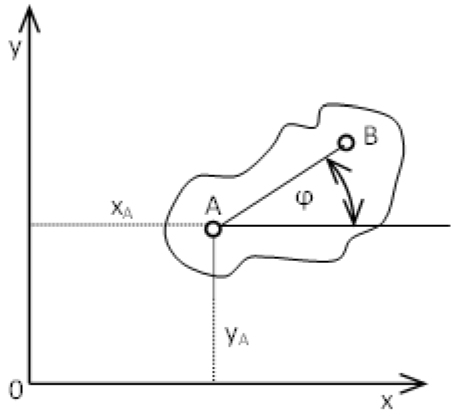


Figure 2.12

A free rigid in the plane can perform three independent movements: movement in the direction of the coordinate axes x and y and rotation around any point (A or B). A rigid body in the plane has three degrees of freedom of movement and its link dependence is expressed as follows:

$$i = v - u = 3 - u \qquad \begin{matrix} > \\ = 0 \\ < \end{matrix}$$

3.4.2. Links of body in the plane

By the individual links, it is possible to remove one, two or three degrees of freedom. In each link a linking reaction can arise, depending on the type of load force system.

Links that leave certain mobility are referred to as kinematic couples. They can be kinematic couples of first (u = 1), or second class (u = 2).

Higher, general kinematic couple removes one degree of freedom (u = 1).

In a (linkage by leaning), one rigid particle leans by its edge against the surface of other particle, while the surfaces of the rigid particles shall be perfectly rigid. Removed possibility of movement and possible reaction are in the common normal (Figure 2.13a).

One degree of freedom of movement is removed by means of another rigid particle, so-called Binary member. Binary member serves as a linking agent between a frame and a particle with which it is connected by rotational or sliding link. It is not loaded by any external forces, only reaction of a particle and frame lying on the same force bearer are acting on it being of the same magnitude and opposite direction. Such binary members, used as links of a particle to a frame are:

- Sliding bearing (Figure 2. 13b) – joint on one side, movable linkage on the other side. The common bearer must pass over the joint. Link must be unilateral or bilateral.
- Bar (Figure 2.13c) – joints on both sides, common bearer is in their link. The link is bilateral, the bar can be drawn or pushed (Figure 2.13d).

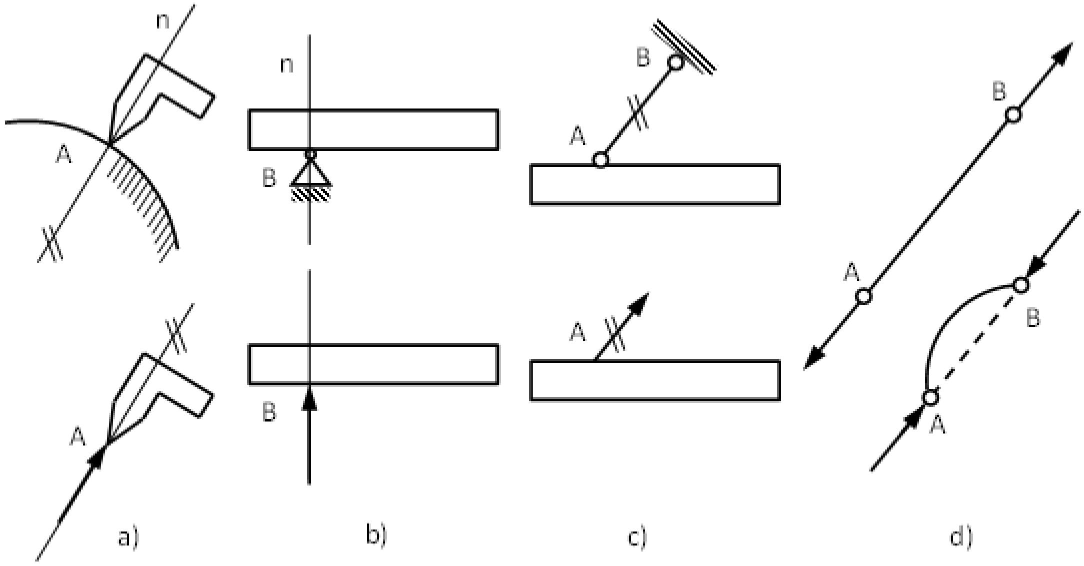
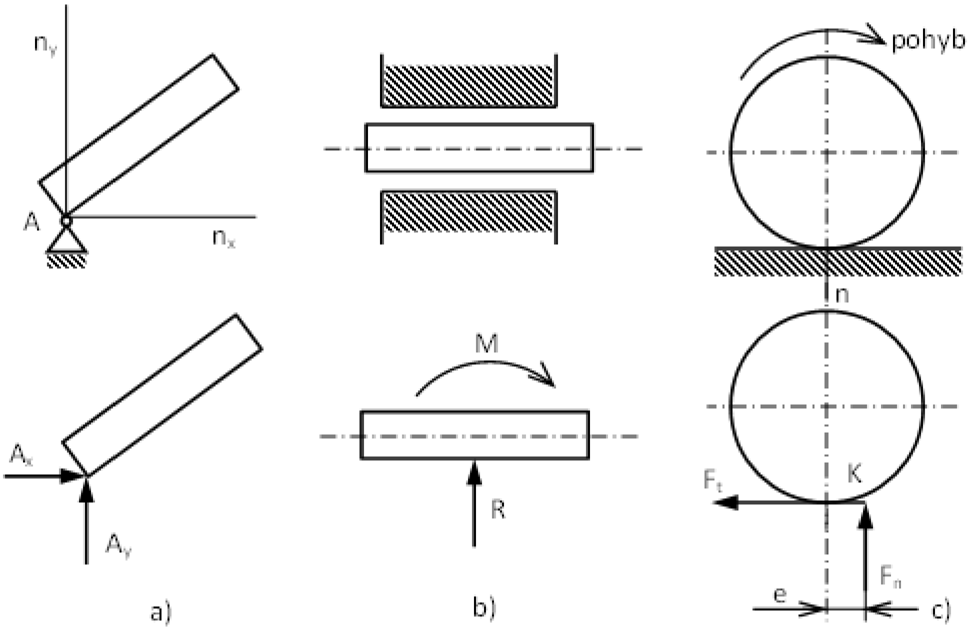


Figure 2.13

Lower kinematic couples (u = 2) remove two degrees of freedom of movement. They include the following links:

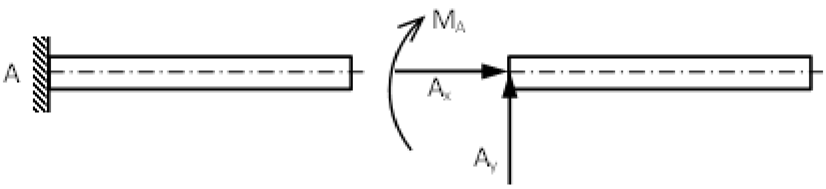
- Rotational link (joint) (Figure 2.14a) removes two movements in the direction of axes x and y. Two parameters are unknown, e.g. magnitude and direction of the reaction. The link leaves the particle a possibility to rotate around the fixed centre of the A rotation.
- Slider link (Figure) leaves a possibility of movement in one direction and removes a possibility of movement in a perpendicular direction and the possibility of rotation in the plane. The unknown parameters are force and moment magnitude.
- Rolling link (Figure) is determined by friction between the particles, removes the possibility of two movements – in the direction of a normal and tangent (reaction F_N a F_T), enables a rolling motion without shear, that is, a possibility of rotation of the particle.

$u = 2, i = 3 - u = 3 - 2 = 1$



A particle can be rigidly fixed with other particle, e.g. with a frame by means of so-called restraint (Figure 3.7), which removes all three degrees of freedom of movement from the particle in the plane.

$u = 3, i = 3 - u = 3 - 3 = 0$

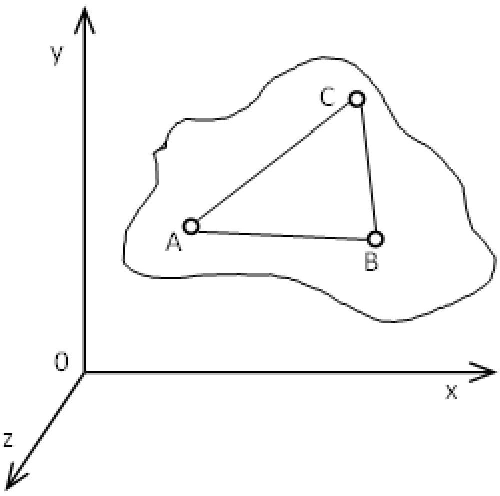


4. RIGID IN SPACE

4.1. Degrees of freedom and linkage dependency of particle in space

The position of a rigid in the space 0, x, y, z is determined by six parameters. These can be 3 coordinates of point A, two coordinates of point B and one coordinate of point C (Figure). Other coordinates of points B and C are linked by fixed distances between the points. For a particle in the space it is v = 6. A free particle in the space has a possibility of six independent movements: movement in the direction of axes x, y and z, and rotation around these three coordinate axes. It has six degrees of freedom of movement and its linkage dependency is as follows:

$$i = v - u = 6 - u \qquad \begin{matrix} > \\ = 0 \\ < \end{matrix}$$

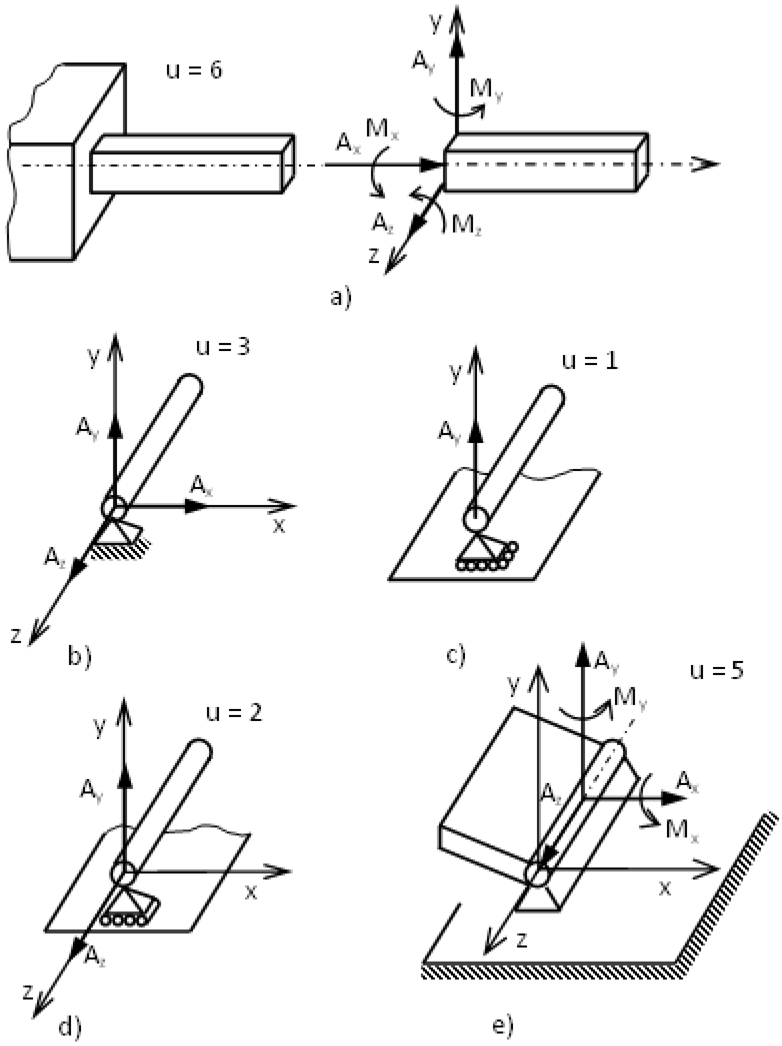


4.2. Links of body in the space

According to the type of link, it is possible to remove 1 – 6 degrees of freedom of movement. The Figure shows individual types of links of a body in the space with marked changes that can occur after loading the body:

- One degree of freedom of movement (u = 1) is removed by link by sliding bearing on rollers (Figure c) and a link by a bar.
- Two degrees of freedom of movement (u = 2) are removed by link by sliding bearing on pins (Figure d).

- Three degrees of freedom of movement (u = 3) are removed by link by spatial joint (Figure b)
- Five degrees of freedom of movement (u = 5) are removed by so-called link by cylindrical joint (Figure e)
- Fixed link – by restraint (Figure a), we remove all six degrees of freedom of movement (u = 6).

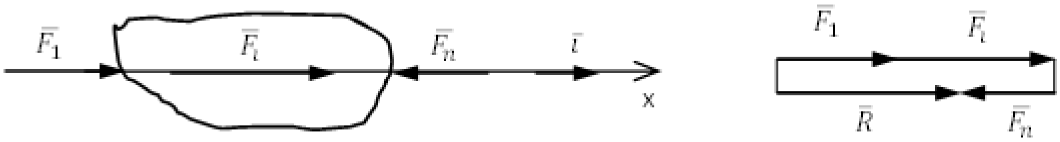


5. COPLANAR FORCE SYSTEMS, PARTICLE EQUILIBRIUM

5.1. Linear force system – LFS

5.1.1. Replacing linear force system

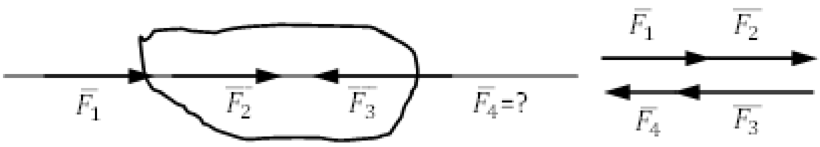
If all forces act in one line on the given object, they can be replaced by one force $\underline{R}$ (resultant), whose line of action is identical with the line of action of all the forces. The axis x with a unit vector $\underline{i}$ represents a line of action of force $\underline{F_i}$ (Figure).



If $\underline{R} = R.\underline{i}$, $\underline{F_i} = F_i.\underline{i}$, then $R.\underline{i} = \sum \quad F_i.\underline{i}$. The resultant magnitude equals to algebraic sum of forces $\underline{R} = \sum \quad \underline{F_i}$.

5.2. Equilibrium of linear force system

The condition of LFS equilibrium is $\underline{R} = \underline{0}$, that is $\sum \quad \underline{F_i} = \underline{0}$.
In the graph, this equation is marked as a closed, so-called SILOVÝ OBRAZEC (Figure).



In analytical solution, the equilibrium condition is expressed in scalar form by one force equilibrium condition $R = 0$, that is $\Sigma Fi = 0$.

LFS equilibrium can also be expressed using condition of moment to any point B (Figure), which does not lie on the line of action of the LFS.

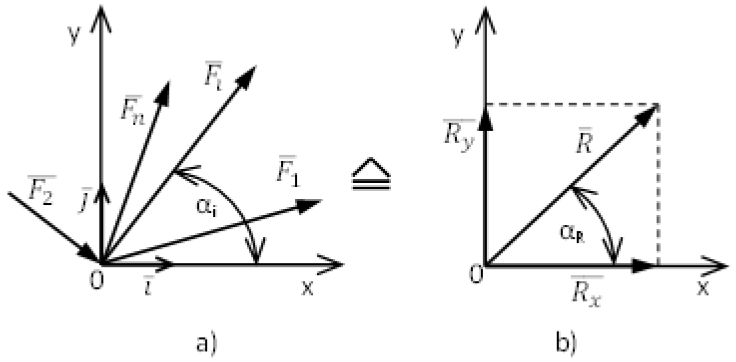
$$\sum \quad M_{iB} = p \sum \quad F_i = 0$$

For linear force system, there is only one independent static condition, that is, from static equilibrium conditions only one unknown parameter can be calculated.

5.3. Central planar force system CPFS

5.3.1. Replacing central planar force system

All forces $\underline{F_i}$ of the given force system pass through point 0 and lie in one plane 0, x, y (Figure a). This way the force system can be replaced by resultant $\underline{R} = \underline{S}$, which passes through point 0 (Figure b).



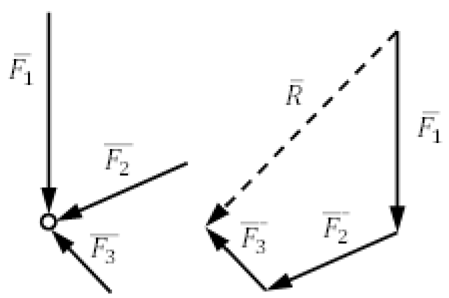
$\underline{R} = \sum \quad \underline{F_i}$

After expressing the force vectors by means of their components, the resultant is $R_x.\underline{i} + R_y.\underline{j} = \sum \quad F_{ix}.\underline{i} + \sum \quad F_{iy}.\underline{j}$.

- If we multiply the equation $R_x.\underline{i} + R_y.\underline{j} = \sum \quad F_{ix}.\underline{i} + \sum \quad F_{iy}.\underline{j}$ by scalar unit vectors $\underline{i}$ and $\underline{j}$, the conditions of replacing CPFS are expressed by two scalar equations: $R_x = \sum \quad F_{ix} = \sum \quad F_i \cos\cos \alpha_i$ and $R_y = \sum \quad F_{iy} = \sum \quad F_i \sin\sin \alpha_i$. The magnitude and direction of the resultant is thus $R = \sqrt{R_x^2 + R_y^2}, \cos\cos \alpha_R = \frac{R_x}{R}$.
- In the graph, the resultant $\underline{R}$ passes through the central of forces 0 (Figure) and it is determined by their vector sum in so-called force pattern.

$\underline{R} = \sum \quad \underline{F_i}$

$\underline{R} = \underline{F_1} + \underline{F_2} + \underline{F_3}$



6.CENTRAL PLANAR FORCE SYSTEM EQUILIBRIUM

The equilibrium condition is $\underline{R} = \underline{0}$, that is $\sum \underline{F_i} = \underline{0}$.

- In analytical solution, two independent equilibrium conditions can be expressed by either two component equations or the component equations can be replaced by momentum (using Varignon´s theorem.

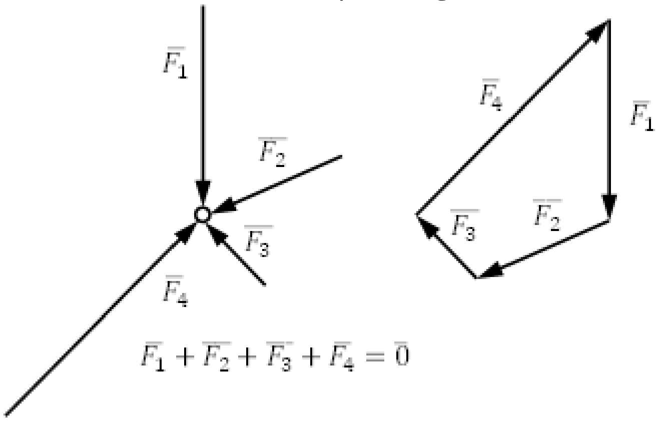
- 1st alternative: component equations
  $R_x = 0$, that is $\sum F_{ix} = 0$
  $R_y = 0$, that is $\sum F_{iy} = 0$
  2nd alternative: momentum equations

  $\left(\sum M_i\right)_A = 0$
  $\left(\sum M_i\right)_B = 0$
  Points A, B and central of forces 0 must not lie on one line!

- 3rd alternative: 1 component and 1 momentum equation
  $\sum F_{ix} = 0$
  $\left(\sum M_i\right)_B = 0$

- Point B must not lie on the axis x! The joining line of central of forces with point B must not be perpendicular to the axis in which the forces are represented.

- In graph solution (Figure) the basis is the condition of closing force pattern with the forces, corresponding to the vector equilibrium condition $\underline{R} = \underline{0}$.

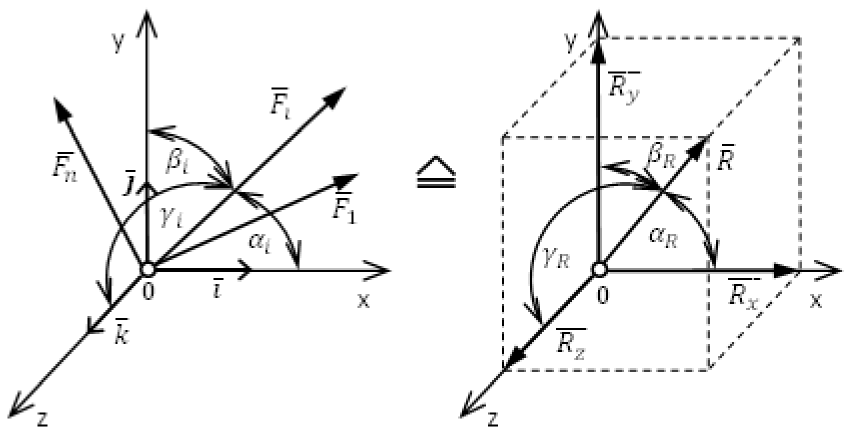


For CPFS only two static conditions can be written from which we can calculate two unknown parameters.

6.1. Central spatial force system – CSFS

6.1.1. Replacing central spatial force system

At one point, central spatial force system n is acting. Each force of this system is determined by its magnitude and direction (Figure a). All forces can be replaced by one force, the resultant $\underline{R}$ (Figure b), which must pass through the common central of forces. Force $\underline{R}$ as a resultant completely replaces the given CSFS.



a) b)

In a rectangular coordinate system 0, x, y, z, the resultant $\underline{R}$ can be divided into the components

$\underline{R_x}$, $\underline{R_y}$, $\underline{R_z}$ (Figure).

$\underline{R} = \underline{R_x} + \underline{R_y} + \underline{R_z}$

The conditions for replacing CSFS are relations for determining the resultant:

$$R_x = \sum \quad F_{ix} = \sum \quad F_i \cos \cos \alpha_i$$
$$R_y = \sum \quad F_{iy} = \sum \quad F_i \cos \cos \beta_i$$
$$R_z = \sum \quad F_{iz} = \sum \quad F_i \cos \cos \gamma_i$$

The resultant magnitude is $R = \sqrt{R_x^2 + R_y^2 + R_z^2}$.

The position of the resultant is calculated as: $\cos \cos \alpha_R = \frac{R_x}{R}$, $\cos \cos \beta_R = \frac{R_y}{R}$, $\left(\cos \cos \gamma_R = \frac{R_z}{R}\right)$.

6.1.2. Central spatial force system equilibrium

CSFS will be balanced if the resultat $\underline{R}$ is zero. (This indicates that the resulting sliding and rotational effect to any point in the space equals zero).

- **1st alternative: component equation**

$\sum F_{ix} = 0$
$\sum F_{iy} = 0$
$\sum F_{iy} = 0$

- **2nd alternative: momentum equations**

On the basis of Varignon´s theorem, the equilibrium conditions can be expressed also by momentum equations with regard to any axes in the space.

$\left(\sum M_i\right)_a = 0$
$\left(\sum M_i\right)_b = 0$
$\left(\sum M_i\right)_c = 0$

Neither of the axes a, b, or c can pass through the common central of the force system, and the axes a, b, c cannot meet in one point or be parallel to each other.

- **3rd alternative: 2 momentum and 1 component equation**

$\left(\sum M_i\right)_a = 0$
$\left(\sum M_i\right)_b = 0$
$\sum F_{ix} = 0$

The axes a, b must not pass through the central of the force system. They cannot intersect in the plane passing through the central of the CSFS and is perpendicular to axis x. The axes a, b must not be parallel if they are at the same time parallel with the axis mentioned above.

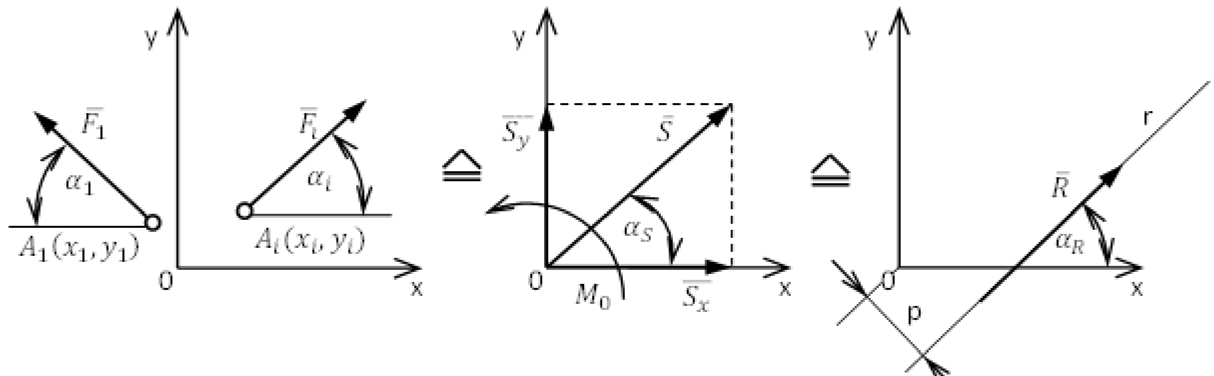
- **4th alternative: 2 component and 1 momentum equation**

$\sum F_{ix} = 0$
$\sum F_{iy} = 0$
$\left(\sum M_i\right)_a = 0$

The axis a cannot pass through the central of the CSFS and must not be perpendicular to the space determined by the axes x, y.
Like in the plane, in the space in terms of analytical solution we always assume the orientation of unknown forces. If the result is positive (+), the orientation/ direction assumed was right, if it is negative (-), the actual orientation of the force is opposite to the assumed one.

7. GENERAL FORCE SYSTEMS. PARALLEL FORCE SYSTEMS. EQUILIBRIUM OF RIGID BODIES

7.1. General planar force system. Analytical solution



a) b) c)

General planar force system consists of forces distributed in the plane (e.g. in plane x, y shown in Figure a). The effect of each force $\underline{F}$ at the beginning of a coordinate system 0, x, y will be both sliding $\underline{F_i}$ and rotational $\underline{M_{i0}}$. The resulting sliding and rotational effect of the force system to the starting point in point 0 will be (Figure):

$$\underline{S} = \sum \underline{F_i}$$
$$\underline{M_0} = \sum \underline{M_{i0}}$$

There can be the following cases:

$\underline{S} \neq \underline{0}$, $\underline{M_0} \neq \underline{0}$ – the system resultant is $\underline{R}$ – not passing the point 0
$\underline{S} \neq \underline{0}$, $\underline{M_0} = \underline{0}$ – the system resultant is $\underline{R}$ – passing through the point 0
$\underline{S} = \underline{0}$, $\underline{M_0} \neq \underline{0}$ – the system is replaced by force couple in plane x, y
$\underline{S} = \underline{0}$, $\underline{M_0} = \underline{0}$ – conditions of GPFS equilibrium

7.2. Replacing GPFS at selected starting point

Magnitude of momentum M_{i0} is expressed using Varignon theorem (Figure a)

$$M_{i0} = x_i F_{iy} - y_i F_{ix}$$

GPFS at the chosen starting point é (Figure b) can be replaced by three scalar equations:

$$S_x = \sum F_{ix} = \sum F_i \cos\cos\alpha_i$$
$$S_y = \sum F_{iy} = \sum F_i \sin\sin\alpha_i$$
$$M_0 = \sum M_{i0} = \sum (x_i F_{iy} - y_i F_{ix}) = \sum F_i (x_i \sin\sin\alpha_i - y_i \cos\cos\alpha_i) = \sum F_i p_i$$

7.3. Replacing GPFS by a resultant

$\underline{S}$ and $\underline{M_0}$ is replaced by the resultant $\underline{R}$, where $R = S$ and $\alpha_S = \alpha_R$, shifted from the starting point 0 by distance p (Figure c). Magnitude of resultant is calculated as follows:

$R = \sqrt{R_x^2 + R_y^2}$, where

$$R_x = S_x = \sum F_{ix}$$
$$R_y = S_y = \sum F_{iy}$$

The angle α_R and position p is calculated as follows

$$\cos\cos\alpha_R = \frac{R_x}{R},\ p = \frac{M_0}{R}$$

7.4. GPFS equilibrium conditions

Equilibrium conditions $\underline{S} = \underline{0}$, $\underline{M_0} = \underline{0}$ is determined by three equilibrium scalar equations:

- 1st alternative: 2 component equations, 1 momentum equation

$$R_x = 0, \quad \sum F_{ix} = 0$$
$$R_y = 0, \quad \Rightarrow \quad \sum F_{iy} = 0$$
$$M_0 = 0, \quad \sum M_{i0} = 0$$

- 2nd alternative: 3 momentum equations

$$\left(\sum M_i\right)_A = 0$$
$$\left(\sum M_i\right)_B = 0$$
$$\left(\sum M_i\right)_C = 0$$

Points A, B, C must not lie in one line, as this line could act as a resultant and the equilibrium conditions would be met.

- 3rd alternative: 2 momentum and 1 component equation

$(\sum M_i)_A = 0$

$(\sum M_i)_B = 0$

$\sum F_{ix} = 0$

The joining line of points A, B must not be perpendicular to axis x (to the axis in which directionwe write the force formula), otherwise the resultant could be on this line and the equilibrium conditions would be met.

8.GENERAL SPATIAL FORCE SYSTEM

General spatial force system consists of forces randomly distributed in the space. It is the most general type of force system, for which it holds true that:

$\underline{S} \neq \underline{0}$, $\underline{M_0} \neq \underline{0}$, $\underline{S}.\underline{M_0} \neq 0$

If in general $\underline{S}$ and M_0 are not perpendicular to each other, GSFS cannot be replaced by one resultant.

8.1. Replacing GSFS at chosen starting point

If a given GSFS at the selected starting point 0, (Figure a), the effect of the i-th force at the point 0 is

- sliding $\underline{F_i}$
- rotational $\underline{M_{i0}} = \underline{r_i} \times \underline{F_i}$

The resulting effect of all forces in the given force system at the point 0 is

- sliding (Figure b) $\underline{S} = \sum \quad \underline{F_i}$
- rotational (Figure c) $\underline{M_0} = \sum \quad \underline{M_{i0}}$

$$\underline{M_0} = \sum \quad \underline{M_{i0}} \sum \quad \underline{r_i} \times \underline{F_i} = \sum \quad \left| \underline{i}\ \underline{j}\ \underline{k}\ x_i\ y_i\ z_i\ F_{ix}\ F_{iy}\ F_{iz} \right|$$

Magnitude and direction of sliding and rotational effect

$$\underline{S} = \underline{S_x} + \underline{S_y} + \underline{S_z} = S_z.\underline{i} + S_y.\underline{j} + S_z.\underline{k}$$
$$\underline{M_0} = \underline{M_x} + \underline{M_y} + \underline{M_z} = M_z.\underline{i} + M_y.\underline{j} + M_z.\underline{k}$$

The effect of GSFS to point 0 is expressed by six equations

$$S_x = \sum \quad F_{ix} = \sum \quad F_i \cos \cos \alpha_i$$
$$S_y = \sum \quad F_{iy} = \sum \quad F_i \cos \cos \beta_i$$
$$S_z = \sum \quad F_{iz} = \sum \quad F_i \cos \cos \gamma_i$$
$$M_x = \sum \quad M_{ix} = \sum \quad (y_i F_{iz} - z_i F_{iy})$$
$$M_y = \sum \quad M_{iy} = \sum \quad (z_i F_{ix} - x_i F_{iz})$$
$$M_x = \sum \quad M_{ix} = \sum \quad (x_i F_{iy} - y_i F_{ix})$$

Magnitude and position of the resulting sliding effect $\underline{S}$ (Figure d) is calculated as follows:

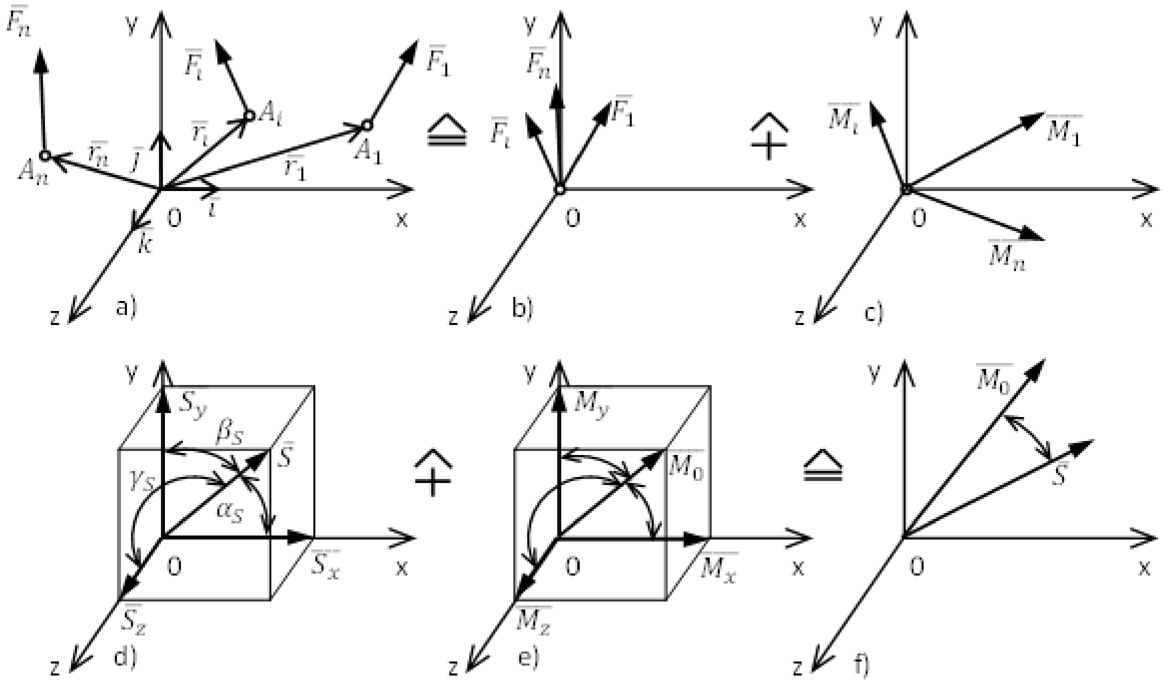
$$S = \sqrt{S_x^2 + S_y^2 + S_z^2}$$

$$cos\ cos\ \alpha_S\ = \frac{S_x}{S},\ cos\ cos\ \beta_S\ = \frac{S_y}{S},\ \left(cos\ cos\ \gamma_S\ = \frac{S_z}{S}\right)$$

Magnitude and position of the resulting rotational effect M_0 (Figure e) is:

$$M_0 = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

$$cos\ cos\ \alpha_M\ = \frac{M_x}{M_0},\ cos\ cos\ \beta_M\ = \frac{M_y}{M_0},\ \left(cos\ cos\ \gamma_M\ = \frac{M_z}{M_0}\right)$$



The angle φ (Figure f) can be determined by means of scalar product

$$\underline{S}\,.\,\underline{M_0} = S\ M_0\ \ cos\ cos\ \varphi$$

where

$$cos\ cos\ \varphi\ = \frac{\underline{S M_0}}{S M_0}$$

8.2. GSFS equilibrium conditions

The conditions of a general spatial force system equilibrium are the following:

$\underline{S} = 0$, $\underline{M_0} = 0$, tj.

$\sum \underline{F_i} = \underline{0}, \quad \sum \underline{M_{i0}} = \underline{0}$

Scalar form: six equilibrium equations:

- 1st alternative: three force and three momentum equations written to coordinate systems

$\sum F_{ix} = 0$

$\sum F_{iy} = 0$

$\sum F_{iz} = 0$

$\sum M_{ix} = 0$

$\sum M_{iy} = 0$

$\sum M_{iz} = 0$

As in the previous chapters, couples can be replaced by momentum equations to other random axes, but the momentum equations cannot be replaced by other force equations. There must be at least three momentum equations to three various random axes. This way we can get other alternatives of expressing equilibrium conditions.

- 2nd alternative: 2 force and 4 momentum equations
- 3rd alternative: 1 force and 5 momentum equations
- 4th alternative: 6 momentum equations to the axes o_1 - o_6

The axes o1 - o6 (does not have to include the axes x, y, z) must not be parallel and must not be intersected by one line.

9. STATIC ANALYSIS OF BODY SYSTEM

A system of bodies is a structure consisting of at least two bodies besides a frame. The individual bodies in a system are linked to each other and to the frame. The bindings which linked the individual bodies to the frame are external links, the links between the bodies are called internal links. According to the type of mutual links, there are systems with different movability and different character of movement of individual bodies. According to characteristic properties the body systems are divided into planar and spatial, movable and immovable, kinematically and statically determinate and indeterminate.

9.1. Kinematic and static determination of planar multi-body systems

Kinematic determination (mobility) of multi-body systems is assessed by means of linkage dependency. The process is analogical to determination of body mobility.

Interlink of two bodies is referred to as a kinematic couple. In the case of planar multi-body systems, kinematic couples are divided by the structure into rotational, sliding and rolling, that remove two degrees of freedom of movement, and general that remove one degree of freedom of movement.

If a system consists of n bodies out of which one was adjusted, that is, was created into a frame, the bodies before their link have

$$v = 3(n' - 1) = 3n$$

Degrees of freedom of movement, where $n = n' - 1$ is a number of bodies besides a frame. If the system contains “m” particles besides “n” bodies, the number of degrees of freedom of such a system before linked is

$$v = 2m + 3n.$$

The overall kinematic determinacy of a system is assessed by the linkage dependency formula

$$i = v - u_{internal} - u_{external}$$

$u_{internal}$ – number of degrees of freedom removed by internal links

$u_{external}$ – number of degrees of freedom removed by external links

When analyzing kinematic determinacy of a system, it is analyzed both internal and external determinacy, where the following situations may occur:

- The overall kinematic determinacy is assessed with regard to internal and external links, where the resulting number of degrees of freedom of system movement is given by the relation

$$i_c = v - u_{internal} - u_{external} \quad \begin{matrix} > & \text{overall kinematically indeterminate} \\ = 0 & \text{overall kinematically determinate} \\ < & \text{overall kinematically overdetermined} \end{matrix}$$

- When analyzing the internal kinematic determinacy only the internal links are considered, that is, the links between the bodies of the system. It is determined by linkage dependency

$$i_c = v - u_{internal} - v_{ST} \quad \begin{matrix} > & \text{internally kinematically indeterminate} \\ = 0 & \text{internally kinematically determinate} \\ < & \text{internally kinematically overdetermined} \end{matrix}$$

where: $u_{internal}$ – number of degrees of freedom of movement removed by internal links

v_{ST} – number of degrees of freedom of a system movement considered as a one rigid body to a frame. For a system considered as a single body in the plane, $v_{ST} = 3$.

- When analyzing the external kinematical determinacy only the external links are considered, which bind the system to the frame, and the system is considered one rigid body. It is determined by linkage dependency

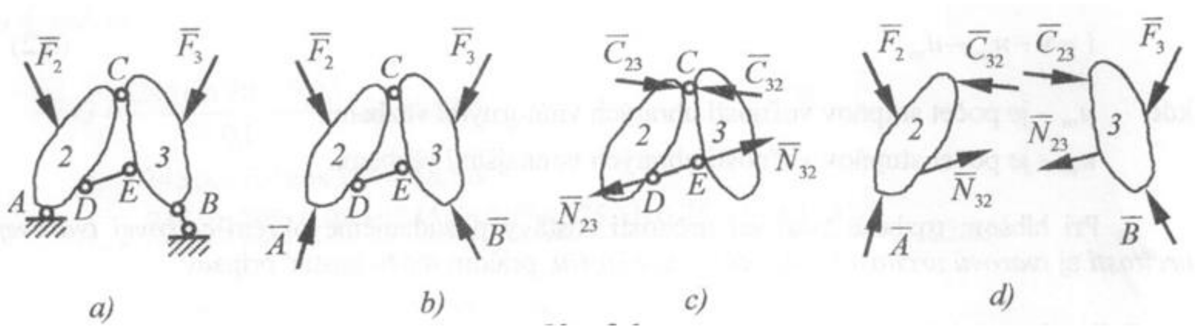
$$i_c = v_{ST} - u_{external} \quad \begin{matrix} > & \text{externally kinematically indeterminate} \\ = 0 & \text{externally kinematically determinate} \\ < & \text{externally kinematically overdetermined} \end{matrix}$$

A system which is kinematically determined can be internally kinematically indeterminate x-times, but at the same time it must be externally k-times kinematically overdetermined.

9.2. Principle of static solution of multi-body systems

The static solution of multi-body systems is based on the theorem of the forces balance. In terms of the forces acting on a multi-body system (Figure a) in its equilibrium, it can be stated that:

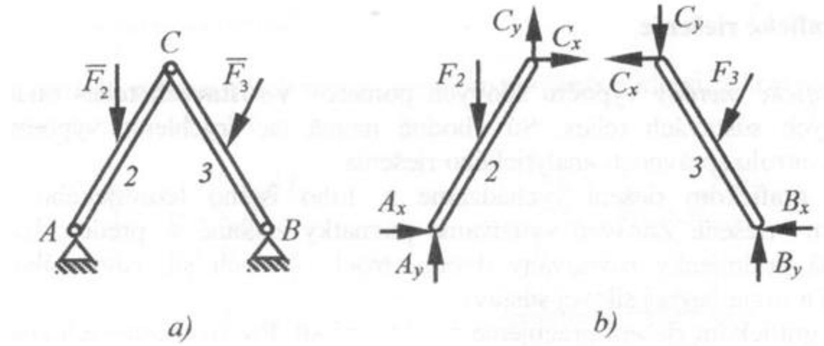
- All external forces (load and reaction) acting on a body in a system are balanced (Figure b)
- In respect to the action and reaction axiom in each link, all internal forces are balanced (Figure c).
- The individual force systems consisting of all forces acting on each body or any set of bodies are balanced (Figure d)



9.3. Analytical (computational) solution of multi-body systems

The basic method of static solution of systems is a release method. The method consists in releasing the individual bodies in the system (sets of member or the whole system) and determining the corresponding equilibrium conditions. If the system consists of “n” bodies without a frame and “m” particles, the equilibrium depends on $r = 3n + 2m$ independent equilibrium equations, based on which in the case of statically determined task the same number of unknown parameters of reactions and additional forces can be calculated.

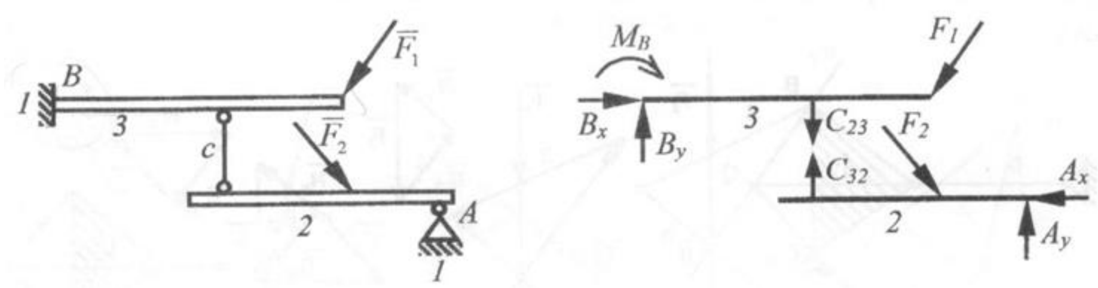
For external forces acting on a system of bodies and for forces action on a certain group of bodies in the system, there are three equilibrium conditions. For example, for calculation or reactions A_X, A_Y, B_X, B_Y, C_X, C_Y 3 and 3 equilibrium equations for released bodies 2 and 3 (Figure b) of the multi-bodies system from Figure a are available.



In respect to their solvability, multi-body systems can be divided into simple and complex.

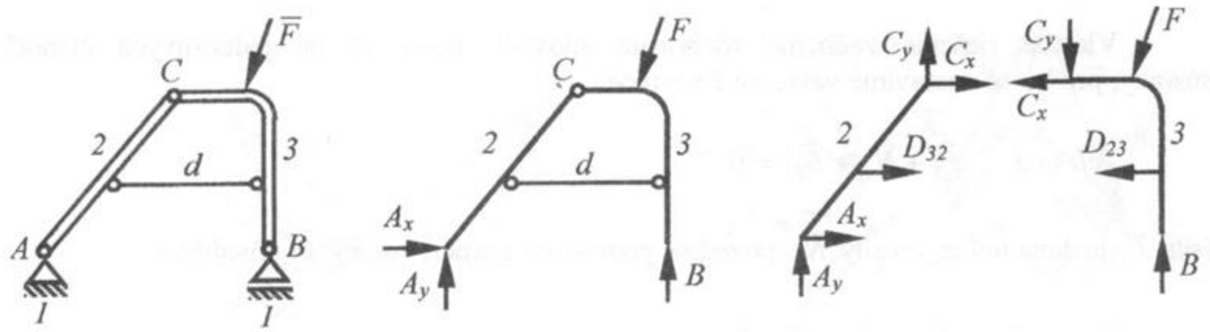
Simple systems

- With $i_{external} \neq 0$ can be solved by gradual solving the equilibrium of its members (Figure).



Based on the body 2 equilibrium, A_X, A_Y, $C_{32} = C_{23}$ will be determined; based on the body 3 equilibrium, 3 B_X, B_Y, M_B can be calculated.

- In the case of $i_{external} = 0$ the system can be solved as a whole and subsequently the individual members equilibrium are solved (Figure)



- Simple systems include systems containing three-hinged arch.
  - Complex systems are those that cannot be solved directly and do not contain three-hinged arch

10. PLANAR BAR SYSTEMS

A specific case of rigid multi-body systems is so-called truss structures, which we encounter in various structures, such as bridges, masts, cranes, roof structures.

According to the spatial arrangement of the structure and the type of external forces system, truss structures are divided into spatial and planar.

Truss structures load can be concentrated at one point (load lifted by a crane) or continuous (weight of a road, weight of the structure itself). In some cases the effect of external load is permanent, while sometimes it changes over time.

To solve truss structures it is necessary to create a suitable static calculation model on the following simplification assumptions:

- Truss structure elements can be considered one-dimensional bodies fixed to the structure by two linkages. These are called binary bodies.
- Linkage of all binary bodies is considered articulated. It is possible also in the case of riveted or welded joints, if the elements connected are not too short. The condition is the arrangement of the elements in individual joints so that the axes of the centers of gravity connected in one joint intersect at one point (called nodal point).
- The structure loading is considered only in nodal points. Continuous load of structure elements is concentrated in two joints which connect the element to the structure.

Such calculation model is called bar system. It is a system of unloaded bodies – bars, which are connected in joints, and they are loaded in the joints. With such load, only axial forces are generated (tense or compression). Bar systems can be spatial or planar.

10.1. Kinematical determinacy of bar systems

Bar system is assessed as a system of particles mutually connected by bars. A bar system can be a whole that is called a bar body.

Kinematical determinacy of a bar body, that is, external kinematical determinacy of planar bar system, will be assessed according to linkage dependency valid for a body in a plane.

$$i_{external} = 3 - u_{external}$$

Internal kinematical determinacy of a bar system is assessed according to

$>$ internally kinematically indeterminate

$i_{internal} = 2s - p$ $= 3$ internally kinematically determinate

$<$ internally kinematically overdetermined

where s – number of joints

p – number of bars

3 – for both relations, the number of degrees of freedom of a bar body

2 – number of degrees of freedom of a free particle in a plane

Overall kinematical determinacy of a bar system

$i_c = 2s - p - u_{vo}$, if $i_c = 0$ – the system is kinematically and statically determined

10.2. Static solution of bar systems

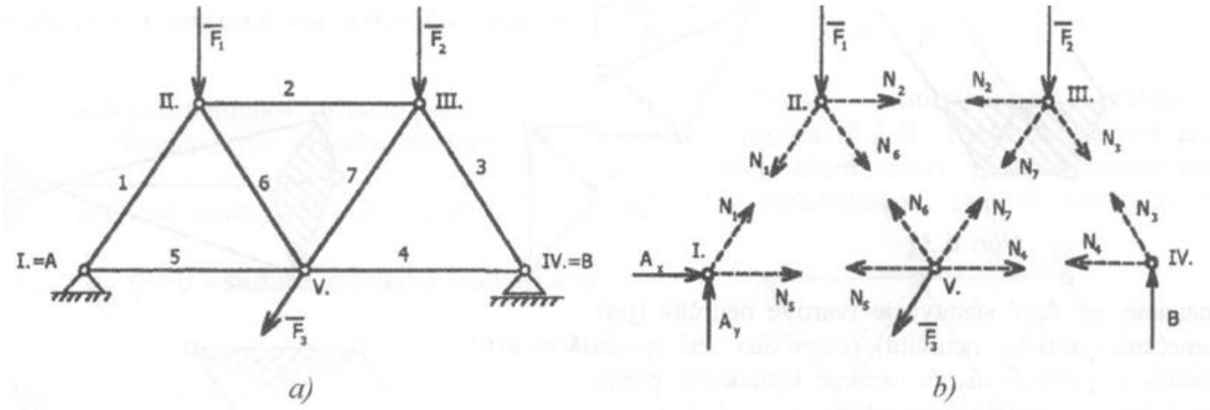
The objective of static solution of bar systems is to identify the magnitude of axial forces in the bars, their orientation and unknown parameters of external reaction depending on external loading force effects. This could be achieved by several methods.

- Central force systems equilibrium that acts only on the individual joints (the method of nodal points)
- Equilibrium of forces acting on a part of a bar system (method of sections)

10.3. Nodal point method

The principle of the method consists in solving the equilibrium of all forces acting on each joint separately. During the gradual releasing of all joints, equilibrium conditions of central force system acting on each joint are determined.

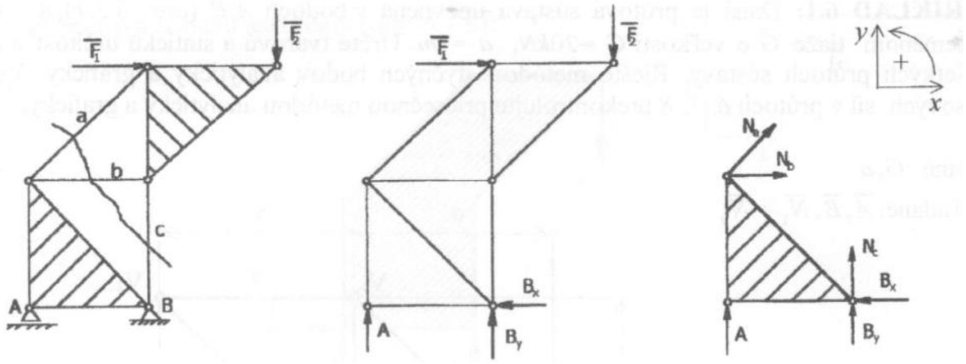
In case that the system as a whole (system body) is kinematically and statically determined, solving equilibrium of external forces (used for calculating external reactions) facilitate the overall solution of equilibrium equations for individual joints.



10.4. Method of sections

Solving axial forces by method of sections is based on the following assumption: if the bar system is balanced, the forces acting on each cut part of the bar system must be balanced. The equilibrium of this part can be solved in the plane or in space. For a body in a plane (space) there can be 3 (6) independent static equilibrium conditions, using which it is possible to determine 3 (6) unknown axial forces. It follows that when using this method it is necessary to divide the bar system by means of imaginary cut over three (6 in space) bars that do not intersect at one point.

From the equilibrium of one of these parts it is possible to calculate axial forces in cut bars. If each of the cut-off parts is subjected to a loading forces and reaction, these reaction must be first determined from the bar body equilibrium.



11. CENTRE OF GRAVITY OF PHYSICAL AND GEOMETRIC OBJECTS

Centre of gravity of a parallel force system with forces linked to A point through which the resultant of this force system passes when turning the system by any angle. If the parallel forces are the Earth gravity forces (the forces of gravity of individual parts of the body), the centre of this force system is called the centre of gravity.

Position of centre of gravity can be determined analytically, graphically and experimentally. In the case of analytic and graphical solution, the condition is the known distribution of the weight in the body, while the graphical solutions focus mostly on planar or symmetric spatial body. Experimental determination of the centre of gravity position is used mostly in the case of complex shapes and non-homogeneous bodies.

11.1. Analytical identification of centre of gravity position

Elementary force $dG = dG = \rho\, dV\, g$ acts on the volume element dV of a body of a weight ρ, while g is a magnitude of gravitation acceleration. For the x-coordinate of the centre of gravity it holds true that

$$x_T = \frac{\int_V \quad x\,\rho\, dV\, g}{\int_V \quad \rho\, dV\, g} = \frac{\int_V \quad x\,\rho\, dV}{\int_V \quad \rho\, dV}$$

while the V volume is fully integrated. Similar equation is true for y_T and z_T. For calculating integrals we must know the distribution of density ρ in the body, that is function $\rho = \rho(x, y, z)$.

In the case of a homogeneous body density or the specific body weight is constant. The centre of the mass in this case is identical with the centre of gravity of the geometric figure. It follows that the position of the centre of gravity of a homogeneous body does not depend on its mass but it´s given by the geometric shape.

$$x_T = \frac{\int_V \quad xdV}{V}, \qquad y_T = \frac{\int_V \quad ydV}{V}, \qquad z_T = \frac{\int_V \quad zdV}{V}$$

In the case of a homogeneous body of a constant thickness *t* (shell) is $dV = t\, dS$, where *dS* is an element of area and the coordinates of such a body are

$$x_T = \frac{\int_S xdS}{S}, \quad y_T = \frac{\int_S ydS}{S}, \quad z_T = \frac{\int_S zdS}{S}$$

where S is overall area of a body, $\int_S x\, dS$ – static moment of a body to the plane *yz.*
In the case of a homogeneous body with a constant cross sectional area S all over its length l is $dV = Sdl$ and the coordinates of such a centre of gravity are

$$x_T = \frac{\int_l xdl}{l}, \quad y_T = \frac{\int_l ydl}{l}, \quad z_T = \frac{\int_l zdl}{l}$$

where l is an element of its length.

If a body can be divided into a specific number of parts whose centers of gravity are known or is possible to calculate, then the centre of gravity of such a composed homogeneous body can be calculated as follows:

$$x_T = \frac{\sum x_i V_i}{\sum V_i}, \quad y_T = \frac{\sum y_i V_i}{\sum V_i}, \quad z_T = \frac{\sum z_i V_i}{\sum V_i}$$

For a shell in the space

$$x_T = \frac{\sum x_i S_i}{\sum S_i}, \quad y_T = \frac{\sum y_i S_i}{\sum S_i}, \quad z_T = \frac{\sum z_i S_i}{\sum S_i}$$

For a body of a constant cross-sectional area, or for a line in the space

$$x_T = \frac{\sum x_i l_i}{\sum l_i}, \quad y_T = \frac{\sum y_i l_i}{\sum l_i}, \quad z_T = \frac{\sum z_i l_i}{\sum l_i}$$

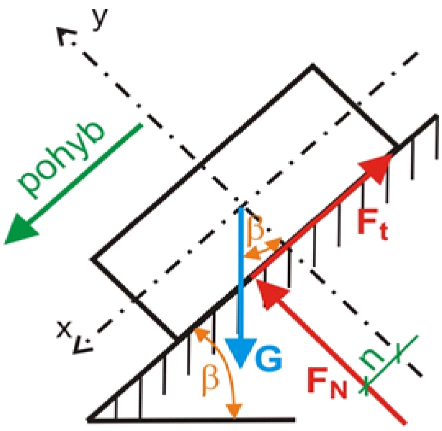
The calculation of the centre of gravity coordinates will be performed by entering the results of sub-calculations in a table.

i	x_i	y_i	z_i	H_i	$x_i H_i$	$y_i H_i$	$z_i H_i$
1 2 ...							
$\sum$				A	B	C	D

Symbol H_i represents one of the variables V_i, S_i, l_i. The coordinates of the centre of gravity will be calculated as a share of the relevant sums.

12. PASSIVE RESISTANCE

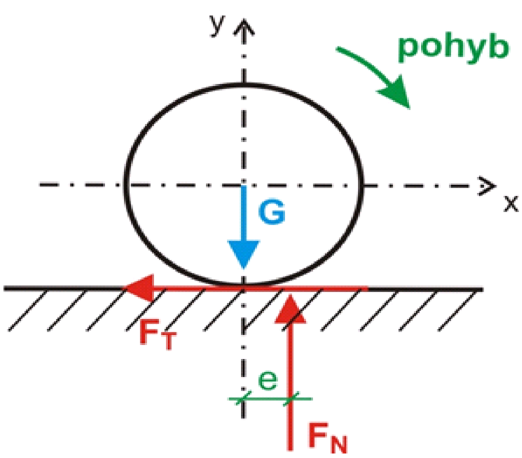
12.1. Sliding friction



Legend: pohyb – movement

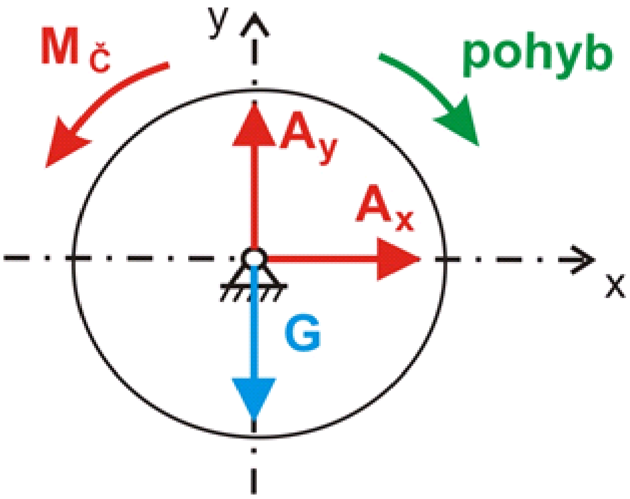
F_t – friction force
F_N – normal reaction
Coulomb´s law: $F_t = F_N \cdot f$
F – coefficient of sliding friction
F_t – friction force - always acts against the movement

12.2. Rolling resistance



F_T – tangential reaction
F_T – act against possible slipping

12.3. Pin friction moment

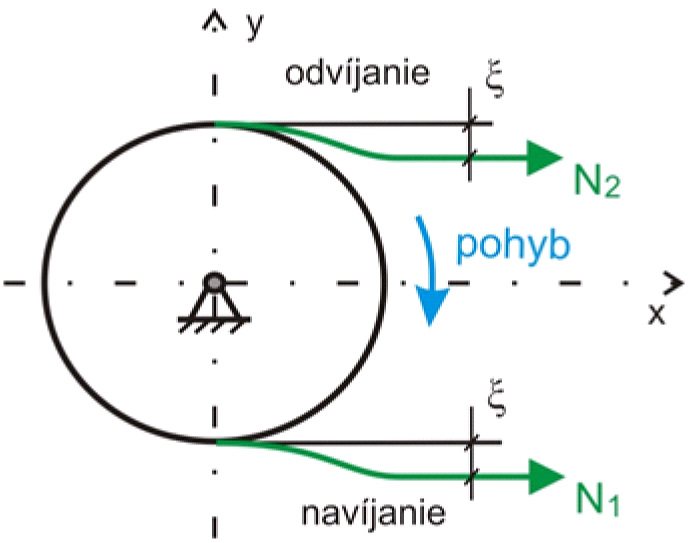


$M_č = f_č . r_č . A$

$M_č = f_č . \, r_č \, . \, \sqrt{A_x^2 + A_y^2}$

Mč – pin friction moment
Fč – coefficient of pin friction
Rč – pin radius
A – resulting reaction in pin

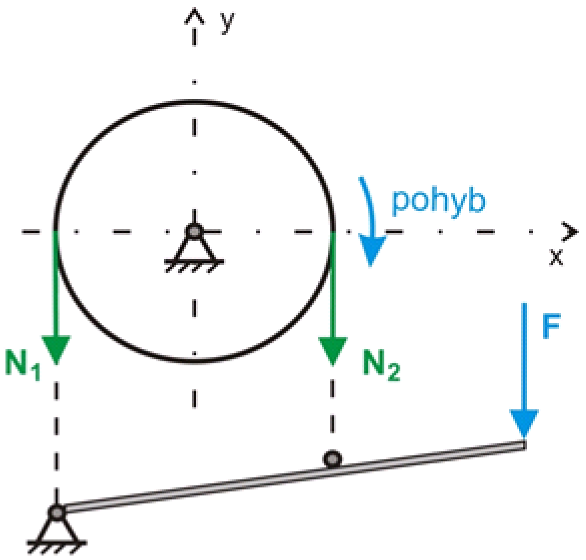
12.4. Rigidity, immobility of ropes



Legend: odvíjení - reeling, navíjení - winding, pohyb - movement

ξ (ksí) – arm of rope rigidity

12.5. Fiber friction on cylindrical surface



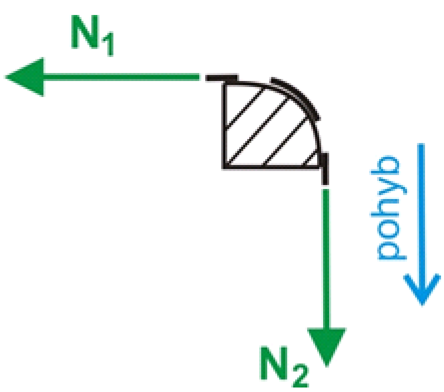
$N_1 > N_2$

Euler´s law: $N_1 = N_2 \, . \, e^{\alpha \, . \, f1}$

α – wrappin angle [rad.]

f_l – coefficient of fiber friction on cylindrical surface

α [*rad*] = **(π / 180) .** α



$N_1 > N_2$

Euler´s law: $N_1 = N_2 \, . \, e^{\alpha \, . \, f1}$